Homework 4

Due: 5pm Friday 12/4/09

1. **Evil aliens:** Suppose evil aliens compressed the Sun so hard that it became a black hole. What would be its radius in kilometers? If evil aliens also compressed the Earth into a black hole, what would be its radius in centimeters? (The Sun's mass is 2×10^{30} kg; the Earth's mass is 3×10^{-6} that of the Sun. The gravitational constant is 6.67×10^{-11} Newton m^2/kg^2.)

2. **How much dark matter is in our solar system?** The *average* mass density of the dark matter in the universe is about 3×10^{-30} g/cm^3. Dark matter is not smoothly distributed in space, however. Instead, it clusters around each galaxy due to gravity. Assume that at our part of the Galaxy, the dark matter is overdense by a factor of 500 above the average density given above. Take the radius of the solar system to be 40 AU (average distance to Pluto). What is the volume of a sphere with this radius in cm^3? What is the mass (in solar masses) enclosed in this sphere? If you were designing the orbits for NASA's upcoming spacecraft to Pluto, should you take into account the dark matter in our solar system in your calculations? Why or why not? [*Handy numbers*: $1AU = 1.5 \times 10^{13}$ cm; $1M_\odot = 2 \times 10^{33}g$.]

3. **Just how much mass is out there?** In class, we discussed how to measure the mass of a galaxy enclosed within a radius from the speed of an orbiting object at that radius. For the following objects orbiting the galactic center, calculate the mass of the Milky Way which is inside each orbit. Your answer should be in units of solar masses. [Hint: Once you get an answer for part (1), don't just repeat the calculations for parts (2) and (3). Instead, it is much easier if you re-scale your answer to (1).]

   (1) The Sun at 8 kpc from the center of the Galaxy, orbiting at a speed of 220 km/s.

   (2) A star at 16 kpc from the center of the Galaxy, orbiting at a speed of 230 km/s.

   (3) A cloud of gas 32 kpc from the center of the Galaxy, orbiting at a speed of 200 km/s.

   (4) Make a plot for the orbital speed (in units of *km/s*) vs. distance from the galactic center (in units of *kpc*) for the Sun, the star, and the gas cloud.

   (5) Make a plot for the mass you calculated (in units of *solar masses*) vs. distance from the galactic center (in units of *kpc*) for the same objects.

   (6) We do not see very many stars or gas clouds between 16 and 32 kpc from the center of the galaxy. What does this tell us about the properties of much of the mass in our Galaxy?

4. **Our too friendly neighbor:** The Andromeda galaxy (M31), is about 800 kpc away. (You don't need a calculator for this problem.)

   (1) If there were only Hubble expansion and no peculiar motion, what would be the speed of Andromeda measured from the Earth? (Use a Hubble constant of 70 km/s/Mpc.)

(2) In reality, Andromeda is moving *towards* us at a speed of of about 120 km/s due to the strong gravitational pull of our Galaxy. If a spectral line is measured to be at a wavelength of 1000.000 nm in the laboratory, what wavelength would astronomers measure for this line when they point their telescope at Andromeda?

(3) Assuming Andromeda's speed remains a constant 120 km/s, how many years would it take for Andromeda to collide with our Galaxy? (Use 1 kpc=3×10^{16} km and 1 year $= 3 \times 10^7$ sec. And yes, you can do this without a calculator.) When this occurs, what evolutionary stage do you think our Sun will be in?

(4) Do you think our solar system will collide with stars in Andromeda? Why or why not?