Astro 7A – Problem Set 1

This problem set should utilize only geometry. Aside from the notion that celestial bodies travel in circular orbits, no knowledge of Kepler's laws of motion is needed or should be used to answer these questions. The point is to appreciate that by using naked-eye astronomical observations, math, physics, and our imagination, we can actually infer the radius of the Earth; then the distance to the Moon; and finally the distance to the Sun, in a logical progression. In this way the ancients climbed the first rungs of the cosmic distance ladder and glimpsed the vastness of the heavens.

1 Syene and Alexandria

(a) On June 21 at noon, in the Middle Eastern city of Syene[1], it is observed that the Sun's reflection can be seen in wells of water dug straight into the ground. The same is not true at that same moment in the city of Alexandria, sitting nearly due north of Syene by about 800 km. In Alexandria, at that time, the Sun casts shadows that are 7.2 degrees from the vertical.

Deduce from this observation the radius of the Earth, $R_\oplus$.

(b) This famous experimental measurement of the size of the Earth assumes that the Sun is "far away." In physics, when someone tells us that something is "far away", the next question we should ask is "far away *compared to what*?" Answer this question: "The distance from the Earth to the Sun is assumed in this calculation to be large compared to . . . ⟨ *fill in the blank* ⟩."

2 From the Earth to the Moon

This problem was adapted from MIT's introductory astrophysics course, as taught by Saul Rappaport to many generations of astronomers.

Check out the composite image of the *lunar* eclipse at

http://antwrp.gsfc.nasa.gov/apod/ap080820.html

To do this problem, you will want to print out this stunning "Astronomy Picture of the Day."

(a) The Earth's shadow is called the umbra. On the picture, draw a circle—with a compass if you have one—that best represents the umbral shadow.

[1]Now Aswan, Egypt.

(b) The center of the Earth's shadow clearly does not lie on the path traveled by the center of the Moon (it's close, though). Explain why not.

(c) Measure the diameter of your circle as well as the diameter of one of the lunar images (in cm).

(d) Deduce from your measurements, and by analyzing Figure 1 below, the radius of the Moon compared to that of the Earth. Be sure to account for the proper geometry of the umbral shadow at the distance of the Moon. In a freak cosmic coincidence, the angular radius of the Moon is nearly identical to the angular radius of the Sun—that is why, in total *solar* eclipses, the Sun appears to be blotted out nearly perfectly. For the purpose of this problem, take the angular radius of the Sun to be 0.25 degrees (i.e., the Sun subtends 0.5 degrees on the sky). Make use of small angle approximations.

(e) Calculate the Earth-Moon distance D in terms of $R_\oplus$.

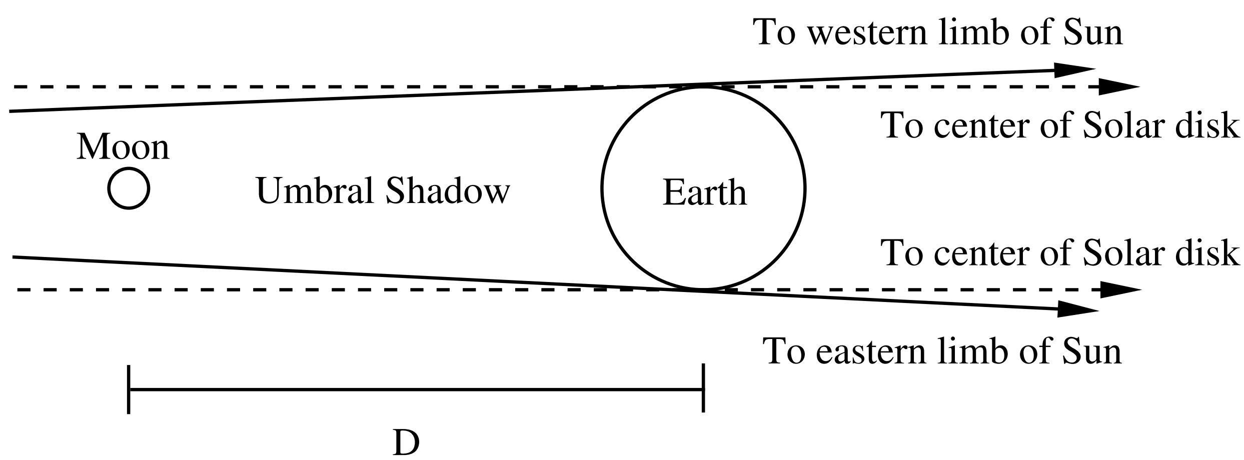

Figure 1: Geometry of a lunar eclipse—not to scale! Note how the umbral shadow is really a cone of darkness that narrows to a point (off the page to the left) because the Sun has a finite size. There are two dashed parallel vectors, both of which claim to point to the center of the Sun. This claim turns out to be justified—we say it is a "good approximation"—insofar as the diameter of the Sun is much larger (about a hundred times larger—but you don't need to know this for the problem) than that of the Earth.

3 The Luminosity of the Sun: $L_\odot$

To an excellent approximation, the Sun can be modeled as a uniformly bright sphere. As such, it emits its radiation *isotropically*—in all directions equally.

(a) On June 21 at noon, a solar panel one square meter in area is laid on the ground in Syene. For simplicity, assume the panel's efficiency is 100%. It is observed to collect 1.4 kW (kilowatt) of power. Deduce from this the total radiant power—a.k.a. the *luminosity*—of the Sun, $L_\odot$. The average distance between the Earth and the Sun (a.k.a the Astronomical Unit) is 1.496 x 10^{13} cm.

(b) Why did this problem need to specify so carefully the exact date and time of the experiment? If I performed this same experiment and used the same formula to derive $L_\odot$ at some other location and time on the Earth's surface, would I get a lower or higher answer for $L_\odot$?