Astro 7A – Problem Set 3

1 Reflection vs. Emission

Kuiper belt objects (KBOs) are detected by virtue of their reflected sunlight. The reflected flux depends on the KBO's *albedo* or reflectivity. Many observations are performed in the V optical passband. Call the V-band albedo a. It ranges from 0 (perfectly absorbing, no reflection) to 1 (perfectly reflecting).

But not all of the detected light is from reflected sunlight. Some of it comes from *thermal emission*—the fact that the KBO is warm. Optical astronomers observing planetary objects usually neglect this thermal emission as small. This problem quantifies "how small is small."

For the problems below, take the V-band luminosity of the Sun to be $L \approx L_\odot$ (of course it's actually less than the bolometric luminosity $L_\odot$, but we are neglecting this difference). Also approximate the KBO as a perfect sphere of radius 100 km; located at a heliocentric distance of 40 AU; observed from Earth at *opposition*;[1] having a V-band albedo $a \approx 0.1$; and emitting as if it were a blackbody. Take the V-band to span wavelengths from 5000 Angstroms to 6000 Angstroms; that is, take the V-band filter to have perfect sensitivity between those wavelengths, and to have zero sensitivity outside those wavelengths.

(a) Estimate the *V-band reflected flux* (not the flux density, but the flux) received at Earth from the KBO, in cgs units. Assume the intercepted sunlight is proportional to the *projected area* of the KBO (loosely speaking, the area perpendicular to the Sun's rays), and assume the reflected sunlight is directed uniformly into 2π steradians (into half the celestial sphere centered on the KBO).

(b) Estimate the *temperature* of the KBO, in Kelvin. Assume the intercepted sunlight is proportional to the projected area of the KBO, and assume the radiation the KBO emits is proportional to the *total surface area* of the KBO, directed uniformly into 4π steradians (into the entire celestial sphere centered on the KBO). Take the KBO to be at a single temperature.

(c) Decide whether the V-band lies on the Rayleigh-Jeans or Wien tail of the KBO's blackbody curve. Depending on your decision, use an appropriately simplified form of the Planck function to estimate the *V-band thermal flux* received at Earth, in cgs units. You will need to do an integral.

(d) Divide (c) by (a) and decide whether it is safe to ignore the thermal contribution to the optical flux.

[1] You can look up this term in either textbook.

2 It's All Relative

Consider again Kuiper Belt Objects (KBOs). They are identified by virtue of their *apparent motion*: they move far more quickly across the sky than do more distant stars and galaxies. You can tell a KBO has moved even during a single night's observing run without any very special equipment.

The apparent motion of a KBO, in general, divides into two components: the KBO's orbital motion around the Sun, and the Earth's orbital motion around the Sun. The former effect is sometimes called the object's *proper motion*, referring to the object's own angular motion in the plane of the sky. The latter effect, due to the Earth's motion, causes *parallax.* This problem decides which contribution is more important.

Consider the KBO to be located at a heliocentric distance of 40 AU, and observed from Earth at opposition. Take both the Earth and the KBO to have circular orbits around the Sun.

Both the KBO and the Earth have tiny masses compared to the Sun, so treat both the KBO and the Earth as test particles orbiting the Sun.

(a) Ignoring the Earth's motion, what would be the proper motion of the KBO as seen from Earth? Express in arcseconds per hour.

(b) Now ignore the KBO's motion. What is the parallax motion of the KBO as seen from Earth? Express in arcseconds per hour.

(c) Some observers have tried to detect KBO apparent motion at *quadrature* (look this up in either textbook). What is the approximate apparent motion under this circumstance?

3 Through A Glass Darkly

If a slab of glass 0.2 m thick absorbs 50% of the light passing through it, how thick must a slab of identical glass be in order to absorb 99.9% of the light?