Astro 7B – Problem Set 1

1 Tidal Friction

Two masses m_1 and m_2 orbit each other with semimajor axis a, with $m_2 \ll m_1$. The orbit is circular. The body m_1 has a rotational moment of inertia I_1 (about an axis that passes through its center of mass) and a spin angular frequency Ω_1. Treat m_2 as a point mass. Take the orbital motion and the spin motion to be in the same direction (but not of the same magnitude).

Consider the tide raised ON m_1 BY m_2 (e.g., the tide ON the Earth raised BY the Moon).

(a) Write down the total angular momentum of the system, L, referred to the axis passing through the center of mass of m_1 (i.e., the rotational axis of m_1), in terms of the variables given. Your expression should have two terms.

(b) As discussed in lecture, when there is tidal friction, the tidal bulge lags or leads the line joining m_1 and m_2, and there is a torque. We call this the "tidal torque". To see how there is a torque, you are encouraged (but not required) to draw pictures of the tidal bulge and how it leads or lags depending on the circumstances (in other words, you can review the lecture material or the textbook).

There is a tidal torque exerted by m_1 on m_2. Likewise, there is an equal and opposite torque exerted by m_2 on m_1. Thus the total torque exerted on both bodies is zero (since there are no external forces acting on the bodies). Therefore the total angular momentum is conserved.

Use $\dot{L} = 0$ to write down an expression for $\dot{a}$ in terms of $\dot{\Omega}_1$. That is, there is a one-to-one correspondence between how quickly the spin changes and how quickly the orbital distance changes.

(c) For this part, take the Earth's $I_1 = 0.3 m_1 R_1^2$.[1] Careful measurements of the Earth's rotation reveal that the Earth's spin period is lengthening at the rate of 1.7 milliseconds per century (i.e., one century from now, the day will last 24 hrs + 1.7 ms).[2] Use this fact and part (b) to infer the rate at which the Moon's orbital distance a is changing, i.e., $\dot{a}$. Express

[1] For spheres of uniform density, $I = 2/5 \cdot mR^2$. The Earth is approximately a sphere, but it is not of uniform density — it gets denser toward the core. Therefore the numerical coefficient of the moment of inertia is less than 2/5; it happens to be about 0.3 for the Earth.

[2] There are several ways this can be measured. The earliest measurements (centuries ago) used timing variations in solar eclipses. Today we can combine precision timekeeping with atomic clocks with precision measurements of distant radio-loud quasars whose positions on the sky are assumed to be fixed in inertial space. (We will study quasars later in this course.)

in cm / yr.[3]

2 Ellipsoidal Light Variations

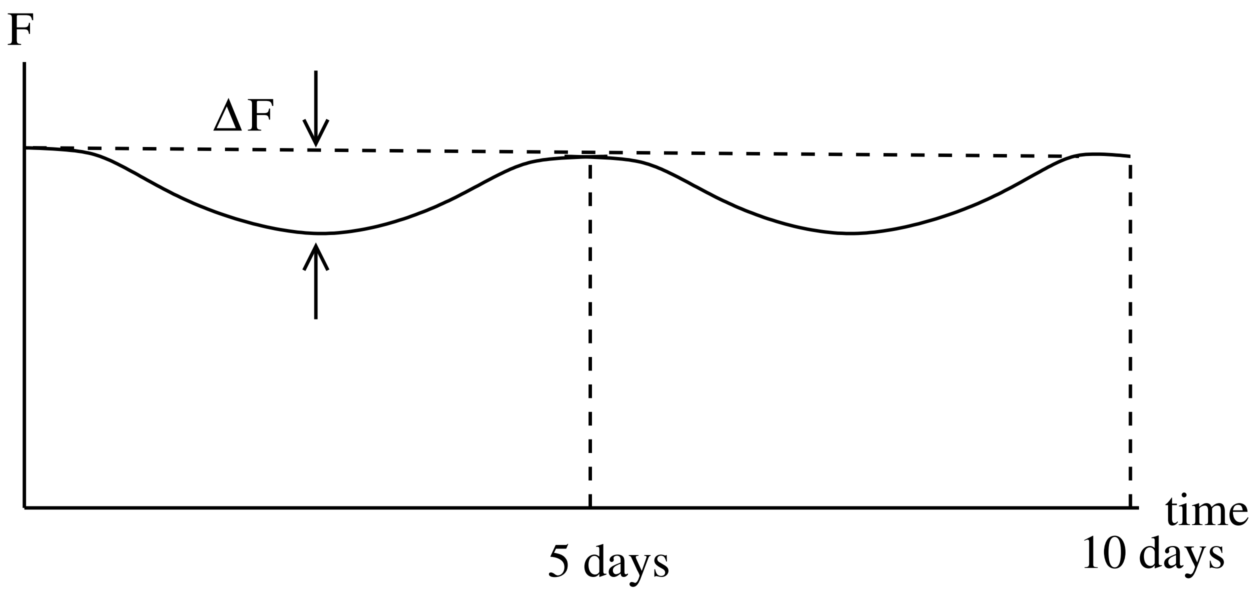

Figure 1: Light curve (flux versus time) for problem on ellipsoidal light variations. The binary is NOT strictly eclipsing; these light variations are due to tidal distortions.

Consider a stellar binary composed of masses m_1 and m_2 separated by semimajor axis a. Mass m_1 is known to be a high-mass star: $m_1 = 30M_\odot$ and its radius $R_1 = 30R_\odot$. The goal of this problem is to figure out m_2 and a.

The light curve of such a binary is shown in Figure 1. From the sinusoidal shape of the light curve it is deduced that the orbit is circular. There are flux variations that appear to repeat every 5 days. The fractional amplitude of the flux variations is $\Delta F/F = 0.03$ (light variations of such a magnitude are easily measurable by, e.g., the *Kepler* spacecraft).

The flux variations are interpreted as "ellipsoidal light variations": variations in the flux emitted by star 1 because the shape of star 1 is tidally distorted by star 2.

Assume that the binary is NOT strictly eclipsing but is nearly so. That is, the orbit is close to edge-on but is not exactly edge-on. From the observer's point of view, the two stars never block one another's light, but they do come close. (We say the binary is "nearly grazing".)

[3]This recession velocity has also been measured directly by shooting lasers at the Moon, bouncing them off mirrors that astronauts left there, and measuring time-of-flight variations.

Assume also that star 1 has a uniform temperature across its surface, and that tidal distortions do not affect this temperature.

(a) What is the orbital period P of the binary?

(b) Derive an *order-of-magnitude* expression for $\Delta F/F$ in terms of m_1, m_2, a, and R_1. For this problem, use the rough formula we derived in class for the height of the tide raised on m_1:

$$h_1 \sim \frac{m_2}{m_1}\left(\frac{R_1}{a}\right)^3 R_1 \tag{1}$$

(c) Estimate m_2 (in solar masses) and a (in AUs). Because our analysis in part (b) is only good to order of magnitude, we cannot expect precise answers in part (c). Full credit will be given for answers that show all work clearly and that give answers that are within a factor of 3 or so from ours.