Astro 7B – Problem Set 8

1 NGC 4258: A Rosetta Stone for the Distance Ladder and Supermassive Black Hole Masses

NGC 4258 is rightly famous for being a galaxy whose (a) distance to the Earth and (b) central supermassive black hole mass are known accurately. This problem shows how both quantities are derived from observations.

The central supermassive black hole happens to be orbited by *water maser clouds*: clumps of water molecules that radiate intensely in a special microwave (radio) line transition. The radio line emission is thought to arise from the outer portions of the black hole's accretion disk, fluorescently excited by X-rays emitted by the inner portion of the accretion disk.

The emitting clouds are concentrated in clumps as shown in Figure 1.

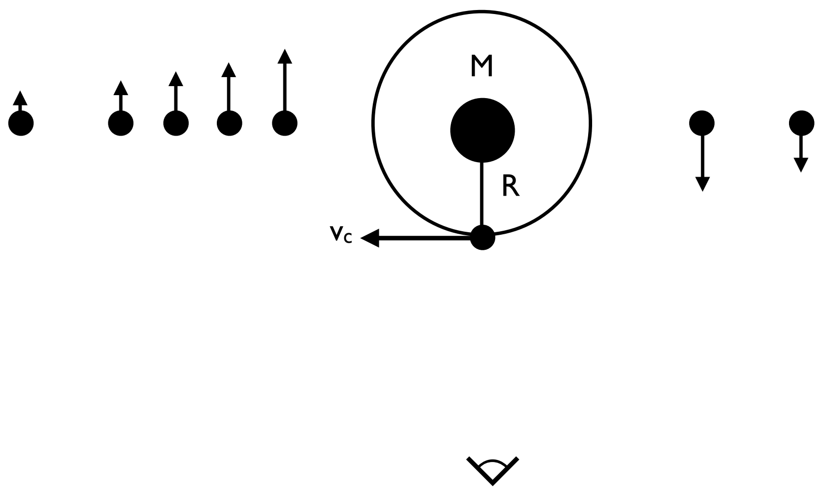

Figure 1: Face-on view of the disk with maser locations shown. The observer on Earth, located at the bottom of the figure, views the disk edge-on.

There are five clumps on one limb (side) of the disk that are rotating away from us, and two clumps on the other limb that are rotating toward us. There is also a clump — actually a bunch of little clumps — that are observed straight-on toward the central black hole. The entire disk is viewed edge-on (actually only nearly so, but we will neglect this for simplicity.[1])

[1] The disk is even slightly warped — which is different from being merely flat and inclined.

The Doppler shifts of the maser clouds are measured precisely. Figure 2 shows the measured radial velocities v_r vs. angular separation on the sky θ from the central black hole:

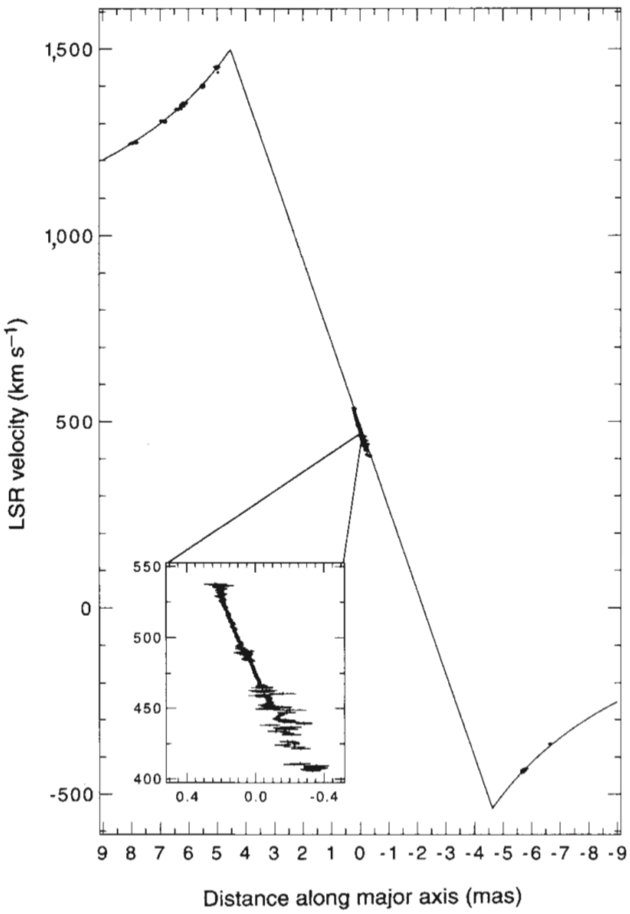

FIG. 3 Line-of-sight velocity versus distance along the major axis (position angle, 86°). Inset, data near the systemic velocity of the galaxy. The position errors are only visible on the scale of the inset. The line is derived from a model whose parameters are listed in Table 1. The high-velocity emission regions define keplerian orbits and constrain the estimate of the enclosed mass. The linear dependence of the systemic emission is a consequence of the change in projection of the rotation velocity.

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Figure 2: Radial velocity versus projected distance from center of NGC 4258. "LSR velocity" refers to the radial velocity relative to the "local standard of rest", which just means the velocity relative to the observer. **In our notation, LSR velocity = v_r.** "Distance along major axis" measures the angular separation on the sky from the central black hole (along the major axis of the accretion disk as seen on the sky), in mas = milli-arcseconds. **In our notation, distance along major axis = θ.**

(a) **Limb clumps: Note the correspondence between the data in Figure 2 and the clumps shown in Figure 1.** The clumps on the limbs are claimed to trace out a Keplerian rotation curve: if true, the clumps should obey Kepler's laws for bodies orbiting a point mass. The central point mass (black hole) should dominate the gravitational potential; the maser clouds should just be test particles by comparison.

Verify that the limb clumps obey a Keplerian rotation curve by measuring from Figure 2 how the radial-velocity-relative-to-the-systemic-velocity $(v_r - v_0)|_{\rm limb}$ varies with angular separation θ: **Write down how $(v_r - v_0)|_{\rm limb}$ scales with θ and verify that the data in Figure 2 supports this proportionality. Here v_r is the full radial velocity, and v_0 is the radial velocity of the system's center-of-mass.**

(b) **Limb clumps: Write down an expression for the central black hole mass M in terms of $(v_r - v_0)|_{\rm limb}$, θ, the distance d between Earth and NGC 4258, and fundamental constants.**

(c) **Central clump:** If the central clump shown in Figure 1 is traveling exactly perpendicular to the observer's line of sight, then $v_r = v_0$.

Say that at time $t = 0$, the central clump exhibits $v_r = v_0$. Observations show that a time $t = \Delta t$ later, the central clump shown in Figure 1 exhibits a new relative velocity $v_r = v_0 + \Delta v_r$.

Assume the central clump executes a circular orbit with radius R and velocity v_c as shown in Figure 1. **Derive an expression for v_c in terms of R, Δv_r and Δt.**

(d) **Central clumps:** In reality, the central clump shown in Figure 1 is actually a bunch of little clumps, distributed along a tiny arc. This is evident in Figure 2, which shows many data points near the center of the plot. These data show that $(v_r - v_0)|_{\rm central} \propto \theta$ (a linear trend). Note that $(v_r - v_0)|_{\rm central} \neq (v_r - v_0)|_{\rm limb}$ (as is clear from Figure 2).

Derive an expression for $(v_r - v_0)|_{\rm central}$ in terms of θ, v_c, R, and d. Work in the small-angle approximation.

(e) **Radio observations show that $\Delta v_r = 9.5$ km/s over $\Delta t = 1$ yr (Haschick et al. 1994; Miyoshi et al. 1995). Deduce from everything you have written down in parts (a)–(d), plus the real data in Figure 2 (Miyoshi et al. 1995), the central black hole mass M (express in $M_\odot$), the distance d (express in Mpc), the velocity v_c (express in km/s), and the radius R.** Congratulations — you have reproduced some of the most famous numbers in all of astrophysics ("bedrock numbers").

2 Galactic Coordinates 10-0-11-0-0-by-0-2 From Galactic Zero Centre

(a) The distance between the Sun and the Galactic Center is 8 kpc. We know this using measurements of *trigonometric parallax (i.e., ordinary parallax)* of the Sagittarius B2 cloud (a point source of radiation emitted near the vicinity of, although technically not quite on, the central supermassive black hole).

Draw the orbit of the Earth around the Sun, assuming the orbit is circular. Draw Sagittarius B2 a distance d away from the Sun. Note that Sagittarius B2 DOES NOT have to be located in the plane of the Earth's orbit (and indeed it is not). Say that the line between the Sun and Sgr B2 makes an angle i relative to the Earth's orbit plane.

What is the maximum apparent shift in the angular position of Sgr B2 over the course of an Earth year? Take $d = 8$ kpc and express in milliarcseconds as a function of i. This is called the "parallactic angle" or simply the "parallax".[2]

Assume for this problem that there are enough telescopes scattered all around the Earth (and even in orbit around the Earth) that Sgr B2 is always visible by some astronomer somewhere.

(b) Another way to measure the distance to the Galactic Center is to exploit the proper motions in Sgr B2. In reality, Sgr B2 is a collection of individual water maser clouds. The clouds are observed to occupy the surface of an EXPANDING spherical shell: presumably the gaseous outflow from a cluster of young stars located near the Galactic Center (imagine a polka-dotted expanding balloon, where each of the polka dots is a water maser cloud). Each of the clouds produces an emission line from a rotational transition in the water molecule. The rest frequency (not the observed frequency) of the water transition is 22.235 GHz (a radio frequency).

Consider the cloud located at the apparent center of the shell as seen on the sky. This cloud is observed to produce a water emission line whose frequency is 22.238 GHz.

Consider a cloud located at the limb (edge) of the shell as seen on the sky. This cloud is observed to have a proper motion of 1 milliarcsec / yr.

Deduce from this information the distance to Sgr B2. Express in kpc. This is a classic example of combining a radial velocity and an angular proper motion to deduce a distance; we applied a variant of this method to find the distance to NGC 4258.

(c) The apparent proper motion of Sagittarius A* (the central supermassive black hole sitting at the Galactic Center) has been measured by radio astronomers to be 6.2 milliarcseconds / yr.

Assume this proper motion is due entirely to the Sun's motion around the Galactic Center. Assume further that the Sun moves on a perfectly circular orbit about the Galactic Center.[3]

[2] For the answer, see Reid et al. 2009, ApJ, 705, 1548, in particular the right panel of their Figure 2, or their Table 4.

[3] In reality, the Sun's motion deviates by a few percent relative to a perfectly circular orbit. We know this by taking an average over all the motions of the stars (relative to the Sun) in the local solar neighborhood.

Deduce from this information the angular velocity ω_0 of the Sun's motion around the Galactic Center. Express ω_0 in units of km/s/kpc.

(d) From your answers in (b) and (c), **calculate the rotational speed Θ_0 of the Sun around the Galactic Center in km/s, and the duration of the Galactic Year in Earth years**.

So actually, the full apparent motion of Sgr A* is 6.4 milliarcsec / yr.