Astro 250 – Planetary Dynamics – Problem Set 1

Try at least 1 out of 3.

Readings: Murray & Dermott 2.3, 2.8 (through eqn 2.122), 2.9, 2.10

Also Binney & Tremaine's *Galactic Dynamics* p.120–124

Problem 1. Apsidal Line Precession

A satellite moves on an elliptical orbit in its planet's equatorial plane. The planet's gravitational potential has the form

$$U \approx \frac{GM_p}{r}\left[1 - J_2\left(\frac{R_p}{r}\right)^2 P_2(\cos\theta)\right] , \quad (1)$$

where r is the distance from the planet to the satellite, θ is the polar angle measured from the planet's spin axis, $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$ is the Legendre polynomial of degree 2, M_p and R_p are the planet's mass and radius, respectively, G is the gravitational constant, and J_2 is a constant that characterizes the dimensionless strength of the quadrupole field of the planet (the degree of planetary oblateness). Celestial mechanicians define their potentials U to be positive, by contrast with the usual convention in physics.

a) Use the appropriate perturbation equation due to Gauss (equation 2.165 of MD) to calculate $\langle\dot{\varpi}\rangle$, the time-averaged precession rate of the satellite's apsidal line.

b) Show that a and e do not suffer any time-averaged variations using the appropriate equations of Gauss (also found in section 2.9 of MD).

Problem 2. The Perversity of Osculating Elements

Deduce the values and time dependences of the osculating elements that characterize a perfectly circular equatorial orbit of radius r around an oblate planet. Employ the potential given by equation (1) above. To get started, compute the relation between the angular velocity and the orbital radius. Remember that the osculating elements are those elements of a Keplerian ellipse that just "kisses" (is instantaneously tangential to) the actual position and velocity of the particle. We fit the Kepler orbit to the motion by *assuming* that the particle moves in a point-mass potential. Here the potential is not that of a point-mass, but we wish to describe the motion of the particle as if it were.

Problem 3. Velocity Ellipsoid in Collisionless Keplerian Disks

Consider a circumstellar disk composed of massless test particles which move without colliding on orbits of eccentricity $e \ll 1$. What is the ratio of the velocity dispersions in the radial and azimuthal directions? Material in the reading from Binney & Tremaine (1987) is relevant to this problem.

By velocity dispersion in an axisymmetric disk we mean the following. Imagine ourselves co-rotating with the disk on a circular orbit. At a given instant in time, we measure the apparent velocities of all particles whizzing by our position. We then (1) square and (2) average the apparent velocities in a particular direction to obtain the squared velocity dispersion, σ^2, in that direction. The problem asks you to obtain the ratio of dispersions in the radial and azimuthal directions, σ_r^2/σ_ϕ^2. Provided the disk is collisionless (particle pass through each other), this ratio is magically independent of the actual distribution of random velocities; i.e., this ratio is independent of the actual distribution of eccentricities, provided they are small.