

## Astro 250 – Planetary Dynamics – Problem Set 6

Try at least 1 out of 3.

Readings: Murray & Dermott 4.1–4.3, 4.5, 4.9–4.11. Chapter 5 will be treated in class, but is not necessary for the following problems.

### Problem 1. Hot Jupiters

About 10 extrasolar, Jovian-mass planets have been discovered in extremely close proximity to their parent, solar-type stars. They have semi-major axes of about 0.05 AU and low eccentricities  $< 0.02$ . They have been referred to as “hot Jupiters.”

a) Consider a Jupiter-mass planet in an orbit of small eccentricity about a solar-mass star. Calculate the semi-major axis,  $a$ , inside of which the eccentricity damping timescale,  $e/\dot{e}$ , is less than  $10^9$  yr (of order the age of the star). Assume that the planet’s  $Q \sim 10^5$  and that  $k_2 \sim 0.5$ . Is it surprising that hot Jupiters have low eccentricity?

b) Astronomers have attempted to detect rings or satellites around HD209458b, a hot Jupiter whose orbit crosses the face of its parent star to generate a detectable, periodic decrease in the stellar intensity; the planet transits the star. The shape of the stellar light curve (intensity vs. time) has been analyzed for the presence of rings or satellites (so far, no luck).

Consider a shepherd moon of a ring system orbiting HD209458b. What is the timescale for its semi-major axis evolution due to tidal interaction,  $a_s/\dot{a}_s$ ? Is it longer than the age of the star? Take the shepherd’s initial semi-major axis to be  $a_s = 2$  planetary radii (inside the Roche zone—see problem 3), the shepherd’s size to be  $R_s = 10$  km (comparable to the sizes of known shepherds in the solar system), and HD209458b to be identical to Jupiter. First decide whether the shepherd’s orbit will decay or expand by making a reasonable assumption about the spin period of HD209458b.

### Problem 2. The Double Synchronous State (problem by Goldreich; but that doesn’t necessarily mean that it’s hard)

Consider an isolated planet-satellite system in which the planet’s spin angular momentum is parallel to the orbital angular momentum. Take the planet to have a constant axial moment of inertia,  $I$ , and approximate the satellite by a point mass. Tidal interactions conserve the total angular momentum,  $L$ , but decrease the total (potential plus kinetic) mechanical energy,  $E$ , of this system. Denote the radius of the relative orbit by  $a$  and the reduced mass by  $m = m_p m_s / (m_p + m_s)$  where  $m_p$  and  $m_s$  are the masses of the planet and satellite.

a) Write down equations for  $L$  and  $E$  in terms of  $m_p$ ,  $m_s$ ,  $m$ , the planet’s spin rate,

$\Omega$ , and the orbital frequency,  $n$ .

b) Show that together  $\dot{L} = 0$  and  $\dot{E} < 0$  imply

$$(n - \Omega)\dot{\Omega} > 0 \quad \text{and} \quad (n - \Omega)\dot{n} > 0.$$

Thus if  $n > \Omega$ , the orbit shrinks and the planet spins up, whereas if  $n < \Omega$ , the orbit expands and the planet spins down.

c) Derive the expression

$$\frac{\dot{\Omega}}{\dot{n}} = \frac{ma^2}{3I}.$$

d) What parameter determines the stability of the state in which  $\Omega = n$ ? By stability we mean whether the system returns to synchronicity if either  $\Omega$  or  $n$  is perturbed off the synchronous state. Estimate the value of this parameter for the Pluto-Charon binary.

### Problem 3. Tidal Disruption and the Roche Zone

This problem examines why ring systems about all the giant planets occupy plane-tocentric distances that are less than  $\sim 2$  planetary radii.

a) Consider a perfectly rigid, spherical satellite of radius  $R_s$ , mass  $m_s$ , and density  $\rho_s$  orbiting a planet of radius  $R_p$ , mass  $m_p$ , and density  $\rho_p$ . Assume the satellite to be in synchronous lock, so that its spin period matches its orbital period. Take the satellite's orbital semi-major axis to be  $a_s$  and its orbital eccentricity to be zero.

A marble rests on the surface of this spinning satellite. The spin of the satellite tries to spin it off. The tidal field of the planet also tries to pull it off. The only force trying to keep it glued to the satellite is the satellite's own gravity. For small enough  $a_s$ , the marble will fly off. What is this minimum semi-major axis,  $a_{s,1}$ ? Express in terms of  $\rho_s$ ,  $\rho_p$ , and  $R_p$ .

b) How does your answer in (a) relate to the radius of the Hill sphere of the satellite,  $r_H = (m_s/3m_p)^{1/3}a_s$ ?

c) Now consider a marble floating on a perfectly strengthless, fluid, synchronously rotating satellite. The satellite's shape is now free to distort because it is sitting in the tidal field of the planet and because it is spinning.

ESTIMATE (no heroics necessary) the semi-major axis of the satellite,  $a_{s,2}$ , inside of which the marble flies off the watery satellite. This is a repeat of (a) except that you will need to account for the distorted shape of the satellite; the satellite has an enhanced

size due not only to the tide raised on it by the planet, but also due to its spin. You should at least decide whether  $a_{s,2}$  should be larger or smaller than  $a_{s,1}$ .

d) At orbital semi-major axes of less than  $\sim 2$  planetary radii, there are no satellites whose sizes exceed 100 km, but there are rings composed of meter-sized boulders and smaller debris, and 10 km-sized satellites. Given your answers in (a) and (c), explain why these observations make sense.