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RESONANT CAPTURE BY INWARD-MIGRATING PLANETS

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ABSTRACT

We investigate resonant capture of small bodies by planets that migrate inward, using analytic arguments and three-body integrations. If the orbits of the planet and the small body are initially circular and coplanar, the small body is captured when it crosses the 2:1 resonance with the planet. As the orbit shrinks, it becomes more eccentric until, by the time its semimajor axis has shrunk by a factor of 4, its eccentricity reaches nearly unity ($1 - e \ll 10^{-4}$). In typical planetary systems, bodies in this high-eccentricity phase are likely to be consumed by the central star. If they can avoid this fate, as migration continues the inclination flips from 0° to $i = 180°$; thereafter the eccentricity declines until the semimajor axis is a factor of 9 smaller than at capture, at which point the small body is released from the 2:1 resonance on a nearly circular retrograde orbit. Small bodies captured into resonance from initially inclined or eccentric orbits can also be ejected from the system, or released from the resonance on highly eccentric polar orbits ($i \simeq 90°$) that are stabilized by a secular resonance. We conclude that migration could drive much of the inner planetesimal disk into the star, and that postmigration multiplanet systems may not be coplanar.

Key words: celestial mechanics — solar system: formation

1. INTRODUCTION

Giant planets are found orbiting nearby stars with semimajor axes as small as 0.04 AU (Schneider 2000).[1] The difficulty of forming planets so close to a star has led to the suggestion that these planets formed at larger radii and migrated inward, perhaps through gravitational interactions with the gaseous protoplanetary disk ("early" migration; Lin, Bodenheimer, & Richardson 1996), or with the planetesimal disk that remains after the gas is dissipated ("late" migration; Murray et al. 1998). We focus here on late migration. When massive planets migrate, they can capture smaller bodies in mean motion resonances. Resonant capture in the solar system was invoked by Goldreich (1965) to explain the frequent occurrence of commensurable periods among planetary satellites; in this case the satellites are migrating outward as a result of tidal friction (see Peale 1999 for a review). Similarly, the outward migration of Neptune probably led to the capture of Pluto and many Kuiper belt objects ("Plutinos") into the 2:3 resonance with Neptune (Malhotra 1993, 1995).

All known examples of resonant capture in the solar system involve outward migration. However, the compact orbits of extrasolar planets are believed to result from inward migration. The purpose of this paper is to explore some of the novel features of resonant capture during inward migration.

We shall use Poincaré elements,

$$\Lambda = (M_* a)^{1/2}\ , \quad \Gamma = \Lambda[1 - (1 - e^2)^{1/2}]\ ,$$
$$Z = \Lambda(1 - e^2)^{1/2}(1 - \cos i)\ ,$$
$$\lambda\ , \quad -\varpi, \quad -\Omega \qquad (1)$$

(e.g., Murray & Dermott 1999); here M_* is the mass of the star in units where the gravitational constant is unity; a, e, and i are respectively the semimajor axis, eccentricity, and inclination; and λ, ϖ and Ω are respectively the mean longitude, longitude of periastron, and longitude of the ascending node. In these variables the Kepler Hamiltonian is

$$H_K(\Lambda) = -\tfrac{1}{2}M_*^2/\Lambda^2\ , \qquad (2)$$

and the mean motion $n = \dot{\lambda} = \partial H_K/\partial\Lambda = M_*^2/\Lambda^3 = (M_*/a^3)^{1/2}$.

For simplicity we shall consider only the case of a test particle that is captured by a massive body on a circular orbit with zero inclination. We shall call this body "Jupiter"; its mass, semimajor axis, and mean motion are denoted by M_J, a_J, and n_J.

2. ANALYTIC RESULTS

The Hamiltonian of the test particle can be written in the form

$$H_K(\Lambda) + \sum_{k,l,m} A_{klm}(\Lambda, \Gamma, Z) \times \cos\,[k\lambda + (l - 2m)\varpi + 2m\Omega - (k + l)\phi_J]\ ; \qquad (3)$$

here $\phi_J(t) = \int n_J(t)dt$ is the azimuth of Jupiter and k, l, and m are integers. The sum of the coefficients of the angles in the argument of the cosine must be zero because the Hamiltonian is invariant to changes in the origin of the azimuth. The restriction that the coefficient of Ω must be an even integer $2m$ arises because the planetary potential is symmetric about the equatorial plane. For low-eccentricity, low-inclination orbits, $|A_{klm}| \propto (M_J/M_*)e^{|l-2m|}\, i^{|2m|}$. The terms with $k + l = 0$ can also contain a contribution from an additional axisymmetric potential, due, for example, to a protoplanetary disk or the equatorial bulge of the central star. The terms with $k = l = 0$ are sometimes called the secular Hamiltonian since they do not depend on either the mean longitude λ or the azimuth of Jupiter, ϕ_J, and thus are time independent for Keplerian motion.

A (p, q) mean motion resonance occurs when $pn \simeq (p + q)n_J$, where $p > 0$ and $q > -p$ are integers. We distinguish inner resonances, which lie inside Jupiter ($a < a_J$, $n > n_J$, $q > 0$), from outer resonances ($a > a_J$, $n < n_J$, $q < 0$).

[1] See http://www.obspm.fr/encycl/encycl.html.

We can isolate the slowly varying resonant angle by a canonical transformation to new momenta and coordinates $(\boldsymbol{R}, \boldsymbol{r})$ defined by the mixed-variable generating function

$$S(\boldsymbol{R}, \ell, g, h) = R_1 \lambda + R_2(\varpi - \Omega) + R_3[p\lambda + q\varpi - (p+q)\phi_J] \; . \quad (4)$$

We have

$$\Lambda = \frac{\partial S}{\partial \lambda} = R_1 + pR_3 \; , \quad \Gamma = -\frac{\partial S}{\partial \varpi} = -R_2 - qR_3 \; ,$$

$$Z = -\frac{\partial S}{\partial \Omega} = R_2 \; , \quad (5)$$

$$r_1 = \frac{\partial S}{\partial R_1} = \lambda \; , \quad r_2 = \frac{\partial S}{\partial R_2} = \varpi - \Omega \; ,$$

$$r_3 = \frac{\partial S}{\partial R_3} = p\lambda + q\varpi - (p+q)\phi_J \; . \quad (6)$$

Near resonance the slowly varying arguments in the Hamiltonian (eq. [3]) have either $k = l = 0$ (secular terms) or $k/l = p/q$ (resonant terms). The most important resonant terms have $k = p$ and $l = q$ (the conclusions below are unchanged for other resonant terms, such as $k = 2p$, $l = 2q$). In this case, in the new variables the resonant argument is just $r_3 - 2mr_2$, and the secular argument is $-2mr_2$. Thus all the slowly varying terms in the Hamiltonian are independent of r_1, so if we average over the fast variable r_1, its conjugate action R_1 is an adiabatic invariant. Therefore

$$C_{pq} \equiv qR_1 = q\Lambda + p\Gamma + pZ = (M_* a)^{1/2}[(p+q) - p(1-e^2)^{1/2} \cos i] \quad (7)$$

is adiabatically invariant.

The averaged Hamiltonian has two degrees of freedom. The nature of the trajectories in the averaged Hamiltonian depends on the relative strength of the secular and the resonant terms. If the secular Hamiltonian is either large or small compared with the resonant Hamiltonian, the timescales for libration or circulation of the associated coordinates r_2 and r_3 are well separated, and the problem can be reduced to one degree of freedom by a second averaging. In the intermediate case, resonances with different m can overlap, leading to chaotic motion. All three regimes are present in various solar system contexts.

Despite these complexities, some of the principal features of resonant capture can be deduced directly from the adiabatic invariant C_{pq} (eq. [7]). Suppose that the test particle is initially on a circular, equatorial orbit at semimajor axis a_i. Then $C_{pq} = (M_* a_i)^{1/2} q$. Once the particle is captured into resonance, its semimajor axis will migrate along with Jupiter's to preserve the mean motion resonance, thereby pumping up the test particle's eccentricity, inclination, or both. The adiabatic invariance of C_{pq} then requires that

$$\kappa \equiv (1-e^2)^{1/2} \cos i = 1 + (q/p)[1 - (a_i/a)^{1/2}] \; . \quad (8)$$

The quantity on the left-hand side is initially unity (since $e = i = 0$) and can only decrease. For an outer resonance ($q < 0$), this requires that $a_i/a < 1$; in other words, capture into an outer resonance can only occur if the perturber migrates outward. If the perturber migrates a long way outward, then $a_i/a \to 0$ and $\kappa \to \kappa_\infty = 1 + q/p$. Note that $0 < \kappa_\infty < 1$ since $p > 0$ and $-p < q < 0$; thus, captured orbits never achieve radial ($e = 1$) or polar ($i = \pi/2$) orbits.

Similarly, capture into an inner resonance ($q > 0$) can only occur if the perturber migrates inward. However, in this case the eccentricity and inclination evolution is more dramatic. The quantity $\kappa \to 0$ when $a = q^2 a_i/(p+q)^2$; in other words, by the time Jupiter has migrated inward by a factor of $q^2/(p+q)^2$ after capture, the test particle has been driven onto either a radial or a polar orbit. If migration continues past this point and the test particle remains in resonance, it must be driven onto a retrograde orbit ($\kappa < 0$). The minimum possible value of κ is -1 (for $e = 0$ and $i = \pi$). This is attained when Jupiter has migrated inward by a factor of $q^2/(2p+q)^2$ since capture; if migration continues past this point, the test particle cannot remain in resonance.

These simple considerations suggest that inward migration of a planet can excite small planets or planetesimals into high-eccentricity, high-inclination, or retrograde orbits. In the remainder of this paper, we shall investigate the evolution and fate of these objects numerically.

3. NUMERICAL INTEGRATIONS

In our integrations we use units in which the gravitational constant and Jupiter's initial semimajor axis a_{Ji} are both unity, and $M_* = 4\pi^2$; thus, Jupiter's initial orbital period is $(1 + M_J/M_*)^{-1/2} \simeq 1$. Jupiter's mass is $M_J = 0.001 M_*$. Jupiter is assumed to follow a circular orbit that migrates inward according to the rule

$$a_J(t) = a_{Jf} + (1 - a_{Jf}) \exp(-t/\tau) \; , \quad (9)$$

where $a_{Jf} = 0.1$ and $\tau = 1000$. The test-particle integrations were carried out in regularized coordinates.

Figure 1 shows the evolution of a test particle initially on a zero-inclination, circular orbit with semimajor axis $a_0 = 0.59$. The test particle is captured into the 2:1 resonance ($p = q = 1$) with Jupiter at $t/\tau = 0.073$. Thereafter its orbit shrinks and its eccentricity grows, as predicted by the adiabatic invariant κ (eq. [8]). Eventually, at $t/\tau = 1.90$, when the semimajor axis has shrunk by a factor of $q^2/(p+q)^2 = 4$ since capture, the orbit is nearly radial ($1 - e \leqslant 10^{-4}$). In typical planetary systems, a small body in this high-eccentricity phase would be vaporized by or collide with the star, but we shall continue to follow the dynamical evolution in case the particle survives. At this point the inclination flips rapidly from 0 to π (recall that inclination is undefined for radial orbits). Once the inclination is π, the eccentricity shrinks as the orbit continues to shrink, again as predicted by adiabatic invariance. Finally, at $t/\tau = 5.40$, when the semimajor axis is a factor of $(1 + 2p/q)^2 = 9$ smaller than at capture, the test particle is released from the resonance on a retrograde, equatorial, nearly circular orbit.

Figure 2 shows the final states of an ensemble of test particles on initially circular, zero-inclination orbits with initial semimajor axes a_i between 0.26 and 0.6. The planet migrates from $a_{Ji} = 1$ to $a_{Jf} = 0.1$, so the 2:1 resonance migrates from $a_{Ri} = 0.6300$ to $a_{Rf} = 0.0630$. According to equation (8), particles will be captured into resonance during the migration if $a_i > a_{Rf}$, flipped onto retrograde orbits if $a_i > (1 + p/q)^2 a_{Rf} = 4a_{Rf} = 0.2520$, and released from resonance on nearly circular retrograde orbits if $a_i > (2p/q + 1)^2 a_{Rf} = 9a_{Rf} = 0.5670$. For particles with $4a_{Rf} < a_i < 9a_{Rf}$, the eccentricity is given by

$$a_i = a_{Rf}[2 + (1 - e_f^2)^{1/2}]^2 \quad (10)$$

(*dotted line*). All these predictions agree very well with the results shown in the figure.

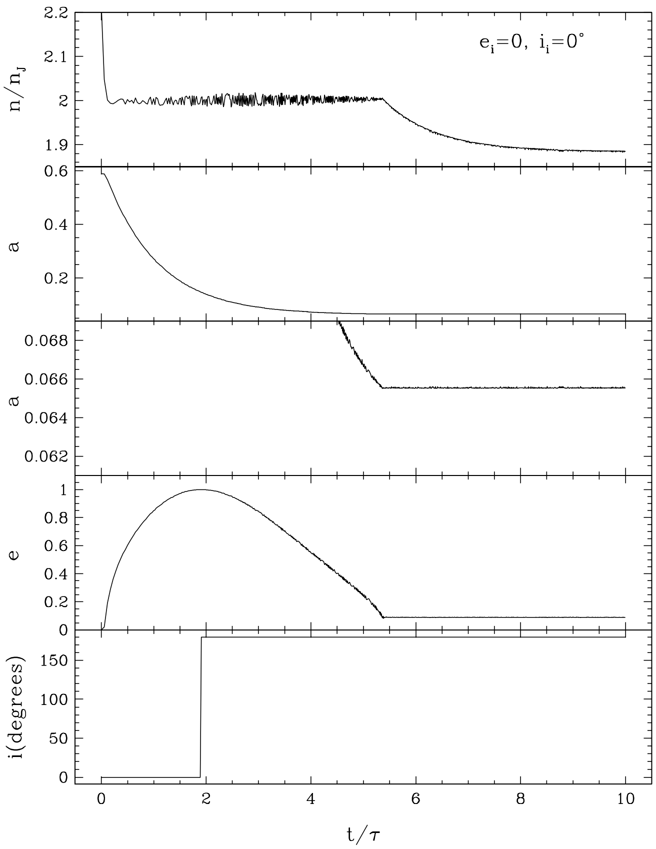

FIG. 1.—Resonant capture of a test particle by an inward-migrating planet. *Top to bottom*, the ratio of the mean motions of Jupiter and the test particle, the semimajor axis (two plots on different scales), the eccentricity, and the inclination of the test particle. The test particle is initially on a circular, equatorial orbit ($e_i = 0$, $i_i = 0°$; Fig. 2, *large open circle*). It is captured into the 2:1 resonance with Jupiter and later released on a retrograde, near-circular orbit ($e = 0.09$, $i = 180°$).

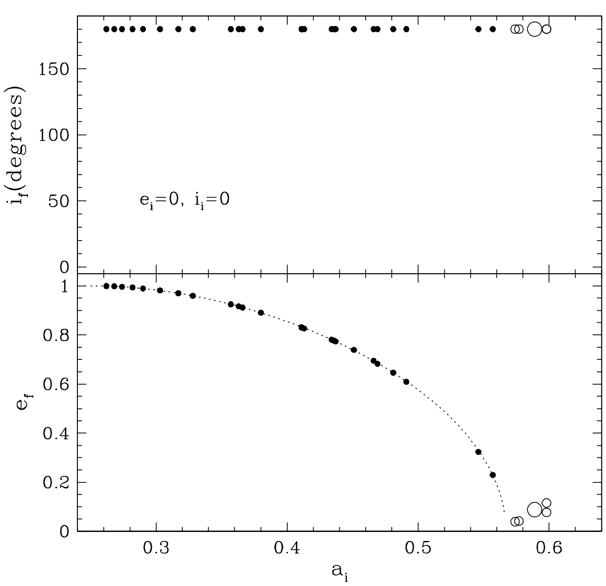

FIG. 2.—Resonant capture of test particles on initially circular, equatorial orbits: final inclination and eccentricity, i_f (*top*) and e_f (*bottom*), as a function of the initial semimajor axis a_i. Filled circles represent test particles that are still in the 2:1 resonance at the end of the integration, while open circles represent particles that have been released from resonance. The dotted line shows the prediction of eq. (10) for the final eccentricity. The large open circle is the test particle whose evolution is plotted in Fig. 1.

The evolution is more complicated when the test particles have nonzero initial eccentricities and inclinations. Figure 3 shows the final states of test particles with the same range of initial semimajor axes as in Figure 2, but with eccentricities and inclinations chosen from the Schwarzschild or Rayleigh distribution,

$$n(e, i)de\,di \propto \exp\left(-\frac{e^2}{2e_0^2} - \frac{i^2}{2i_0^2}\right)e\,de\,i\,di\,. \quad (11)$$

At the end of the integration, most of the particles are found in one of three final states:

1. The particles shown as filled circles are in retrograde ($i \simeq \pi$) orbits trapped in the 2:1 resonance, like those shown in Figure 2. If migration had continued, these would eventually have been released from resonance on nearly circular orbits, after their semimajor axes had migrated by a factor of 9 since capture (this has already happened for two particles; *open circles*). These particles typically have small initial eccentricities and inclinations, $e_i \lesssim 0.05$ and $i_i \lesssim 5°$.
2. The particles marked by crosses are on escape orbits, or chaotic orbits that are likely to escape in the future. In most cases these particles are pumped onto high-eccentricity orbits, released from resonance, and then suffer a close encounter with Jupiter, leading to a random walk in semimajor axis and eventual escape. The evolution of the orbital elements in a typical case is shown in Figure 4. These particles typically have initial eccentricities $e_i \gtrsim 0.05$. Note that the final eccentricity and inclination for the escaping orbits are given in the center-of-mass frame, while all other orbital elements in this and the other figures are given in the star-centered frame.
3. The particles marked by triangles were trapped in the 2:1 resonance and later released on nearly polar orbits ($i \simeq \pi/2$). These particles have high eccentricities and arguments of perihelion ω that librate around $\pm\pi/2$. This secular resonance protects high-eccentricity orbits from close encounters with Jupiter, so the particle orbits are stable on the timescale of our integrations (1×10^4 initial orbital periods of Jupiter). These orbits typically have initial inclination $i_i \gtrsim 5°$. An example is shown in Figure 5.

4. DISCUSSION

Giant planets are found orbiting close to many nearby stars. Because the conditions in the inner parts of protoplanetary disks are unfavorable to planet formation, it is likely that these planets formed at much larger radii and migrated inward. Moreover, because the characteristic evolution times in a protoplanetary disk are longer at larger radius, it is likely that planetesimals and small planets had already formed interior to this giant planet before it began its inward journey. In these circumstances, resonant capture by the inward-migrating planet is likely to occur at the 2:1 mean motion resonance.

We have shown that small bodies that are captured from circular, equatorial orbits into the 2:1 resonance will be excited onto nearly radial orbits by the time their semimajor axis has shrunk by a factor of 4, and released from the resonance on nearly circular retrograde orbits when the semimajor axis has shrunk by a factor of 9. In most of the known planetary systems, these small bodies would be consumed by the central star during this high-eccentricity

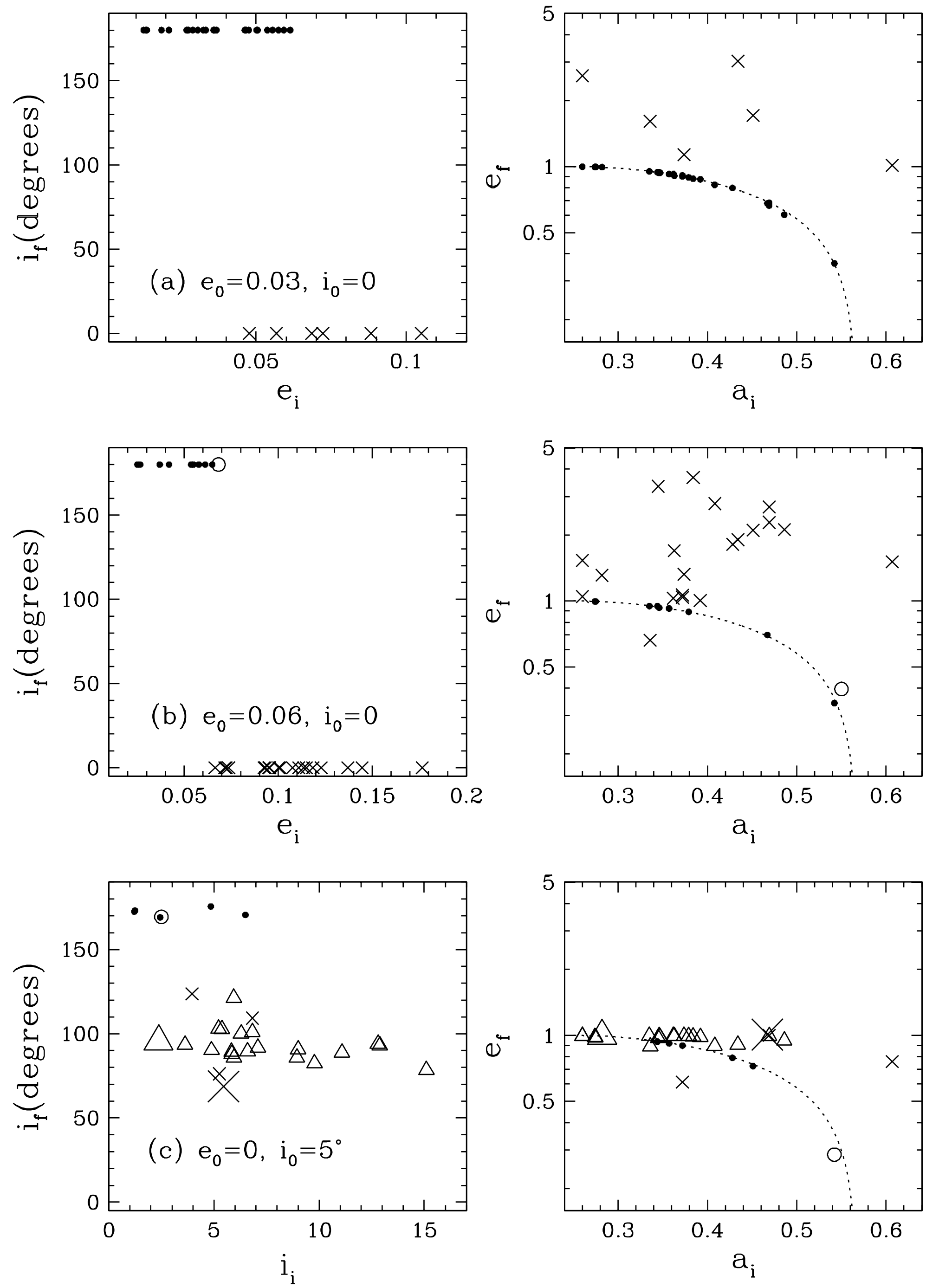

FIG. 3.—Evolution of test particles from low-eccentricity, low-inclination orbits. The initial eccentricities and inclinations are chosen from the Schwarzschild distribution (eq. [11]), with characteristic widths $e_0 = 0.03$, $i_0 = 0°$ (*top*), $e_0 = 0.06$, $i_0 = 0°$ (*middle*), and $e_0 = 0$, $i_0 = 5°$ (*bottom*). The left and right columns respectively show the final inclination and eccentricity as a function of initial eccentricity or inclination and initial semimajor axis. The dotted lines in the right panels show the prediction of eq. (10). The symbols denote the final state of the test particles at the end of the integration: *circles*, retrograde orbits ($i \simeq \pi$); *triangles*, nearly polar orbits ($i \simeq \pi/2$); *crosses*, escape orbits or chaotic orbits that are likely to escape in the future. Filled symbols denote particles that are in the 2:1 resonance at the end of the integration. Eccentricities and inclinations are computed in the star-centered frame, except for escape orbits, for which the elements are computed in the center-of-mass frame. The large symbols in the bottom panels are the test particles whose evolution is plotted in Figs. 4 and 5.

phase. Thus, substantial migration of a giant planet may pollute the outer parts of the star with most of the mass in the interior planetesimal disk: specifically, if the planet migrates from a_{Ji} to $a_{Jf} < a_{Ji}/4$, then the planetesimals with initial semimajor axes in the range $2.520 a_{Jf} < a < 0.630 a_{Ji}$ will be excited to high-eccentricity orbits and are likely to be vaporized by or collide with the central star.

Bodies that do not collide with the star will eventually be released from resonance as migration continues. Some of these will be ejected, but some are protected from ejection by secular resonance and can survive on polar, high-eccentricity orbits. Others are released from resonance on circular and equatorial but retrograde orbits and will survive if the migration stops before the giant planet reaches

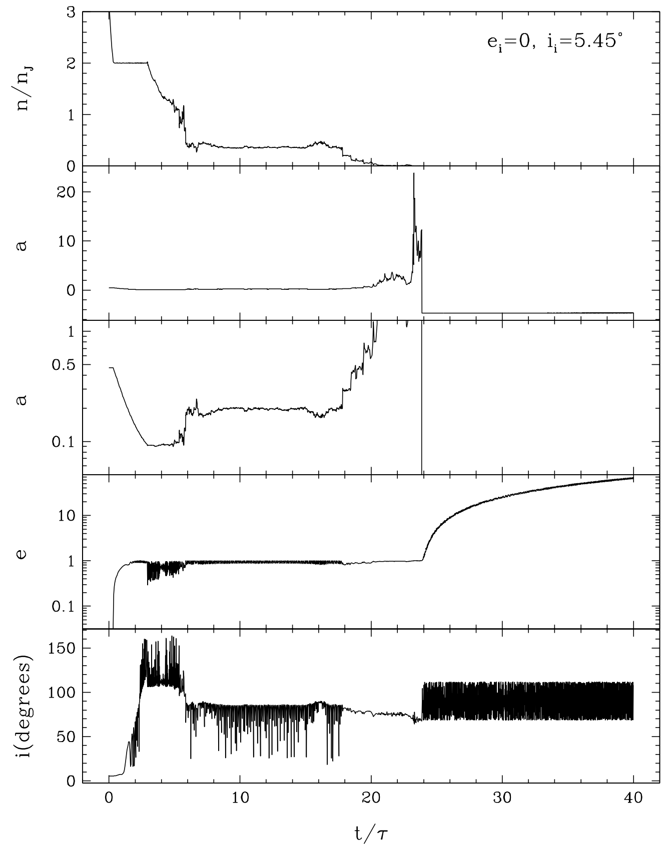

FIG. 4.—Evolution of a test particle with initial eccentricity and inclination $e_i = 0$ and $i_i = 5°.45$ (Fig. 3, *large cross*). *Top to bottom*, the ratio of the mean motions of Jupiter and the test particle, the semimajor axis (two plots on different scales), the eccentricity, and the inclination of the test particle. The particle is temporarily captured into the 2:1 resonance and later released on a chaotic orbit, which eventually escapes after a close encounter with Jupiter. The eccentricity and inclination are not constant at large time, because we are working in a frame centered on the star rather than the center of mass of the star and Jupiter. The final eccentricity and inclination in the center-of-mass frame are $e = 1.01$ and $i = 69°$.

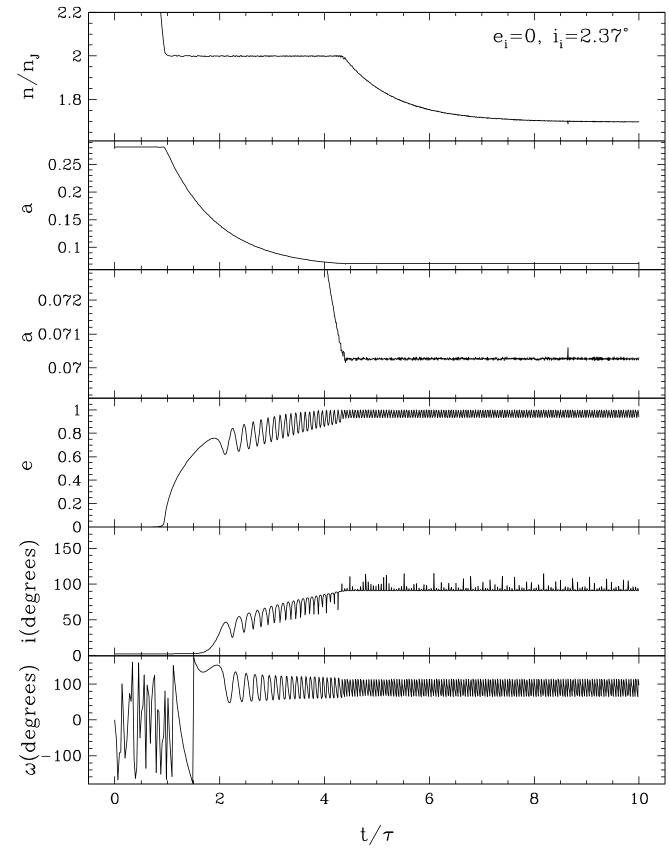

FIG. 5.—Evolution of a test particle with initial eccentricity and inclination $e = 0$ and $i = 2°.37$ (Fig. 3, *large triangle*). *Top to bottom*, the ratio of the mean motions of Jupiter and the test particle, the semimajor axis (two plots on different scales), the eccentricity, the inclination, and the argument of periastron (the angle in the orbital plane from the equatorial plane to the periastron). The test particle is captured into the 2:1 resonance and later released on a polar ($i \simeq \pi/2$) orbit. The particle is protected from close encounters with the planet by a secular resonance, in which the argument of periastron librates around $\pi/2$.

their semimajor axis. Thus, if the captured planets survive the high-eccentricity phase, their postmigration configuration may not be planar; even if the orbits are coplanar, they may not all orbit in the same direction. If multiple bodies that formed from a single disk ("planets") are not necessarily on coplanar orbits, they may be difficult to distinguish from bodies formed from fragmentation of a collapsing gas cloud ("stars"). Radial velocity observations cannot directly measure the relative inclinations of planets in multiplanet systems; this determination requires astrometric measurements such as those planned for the *Space Interferometry Mission.*

Other authors have explored the excitation of small bodies onto high-eccentricity orbits by mean motion resonances (see Quillen & Holman 2000 and references therein). However, most of these investigations have focused on resonances associated with eccentric planets ($|q| > 1$), which can excite eccentricity without migration; thus, migration is only important as a mechanism for capturing particles into resonance.

This preliminary study has not addressed many important issues, including the effects of an eccentric planetary orbit, the stability of these resonances to gravitational noise from other planets and massive planetesimals, and migration in systems containing two or more giant planets.

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