Astrophysical Fluid Dynamics – Problem Set 10

Readings: Course Reader: selections from Binney & Tremaine. Also Shu pages 98–101

Problem 1. Inner Disk Edge

A gas disk orbits an object of mass M. Disk self-gravity is negligible. The inner edge of the disk is located at radius a. Gas there has a sound speed c_s and an orbital angular frequency Ω.

The inner edge is not perfectly sharp. At the inner edge, the gas pressure decreases (going inward) over a radial length scale δ. Make an order-of-magnitude estimate for the smallest δ can be before gas at the disk edge becomes Rayleigh-unstable.

Problem 2. Clumping Sound Waves

Shu Problem Set 3, problem 1abc.

NB: The previous generation of 202 students complained that they did not get much out of Shu's problem sets because they were excessively technical. I have considered this complaint, but still feel that certain problems are worthwhile. This one is worthwhile because it offers a situation to study gravitational instability without having to pull the Jeans swindle, and it also provides practice in deriving dispersion relations.

Problem 3. Magnetospheres

(a) Estimate the radius r_A of the Earth's magnetosphere, in units of the Earth's radius.

(b) Estimate the radius r_A of an accreting T Tauri star's magnetosphere, assuming $\delta \sim r$ (see lecture notes).

Use $M_* \sim 1 M_\odot$, $R_* \sim 3 R_\odot$, a dipole field of surface strength $B_* \sim 1$ kG, and the median measured accretion rate of $\dot{M} \sim 10^{-8} M_\odot \, \mathrm{yr}^{-1}$.

Hot Jupiters are speculated to be deposited at $r_J \approx 0.6 r_A$ (at this location, the hot Jupiter is in 2:1 resonance with the disk edge at r_A). Compare your estimate for r_J with the observed radius for a hot Jupiter orbit (0.05 AU).[1]

[1] Given the crudeness of the theory, this comparison is not expected to yield better than order-of-magnitude agreement.