Astrophysical Fluid Dynamics – Problem Set 2

Readings: Pringle & King 1.3–1.4, 1.6.1, 1.7.1, 1.7.3; Shu pages 20–24, 30–33, 49–50; Tritton 10.4–10.7; Landau and Lifshitz Chapter 1, section 5, 6; Movies: e.g., Eulerian/Lagrangian, Fluid Quantity and Flow, Flow Visualization

Problem 1. The Many Guises of the Energy Equation

Start from the *total energy equation* (Shu equation 2.32; we derived this in class):

$$\frac{\partial}{\partial t}\left(\frac{\rho}{2}|u|^2 + \rho\varepsilon\right) + \frac{\partial}{\partial x_k}\left[\frac{\rho}{2}|u|^2 u_k + u_i\left(P\delta_{ik} - \tau_{ik}\right) + \rho\varepsilon u_k + F_k\right] = -\rho u_k \frac{\partial \phi}{\partial x_k} \qquad (1)$$

and derive the *internal energy equation* (Shu equation 2.36):

$$\rho\frac{D\varepsilon}{Dt} = -P\nabla\cdot\vec{u} - \nabla\cdot\vec{F}_{\rm con} + \tau_{ik}\frac{\partial u_i}{\partial x_k} \qquad (2)$$

The notation above follows that in lecture.

Problem 2. Stupid Human Tricks

In grade school[1] my friends and I[2] used to suck soda up a straw, cap the top end of the straw with a finger, and watch with amazement as the soda stayed in the straw, even when we removed the straw from the glass.

Explain this phenomenon. What exactly is pushing against the bottom of the straw to keep the soda from falling? What is the tallest straw with which you can perform this trick?

Problem 3. Torricelli's Tank or, "A Hole in My Bucket"

A tall cylindrical tank having radius R_1 is filled with water. A nozzle of radius $R_2 \ll R_1$ is opened at the bottom of the tank, and water comes pouring out. Take the height of water in the tank to be h at some time.

(a) Derive an expression for the velocity with which water emerges from the nozzle, assuming the flow is steady and inviscid.

[1] And perhaps beyond
[2] Or maybe it was just me

(b) The assumption of steady flow is not justified over arbitrarily long times, since the water level is continuously dropping. Estimate the timescale over which the flow can be approximated as steady.