Order-of-Magnitude Physics – Solution Set 1

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1 Put Your Money Where Your Mouth Is

Estimate the sizes (in dollars) of the following industries in the United States:

- *K–12 Education*
- *Entertainment (excluding sports)*
- *Sports*
- *Restaurant*

K-12 Education

Let's start by answering an easier, related question: how many K-12 students are there in the United States? All States make school attendance mandatory up until a certain age, and most students attend for the full 13 years. So, let's make the simplifying assumption that *all* children aged 5-18 attend school. How many people is that? Well, we don't know the exact age distribution, so let's make another simplifying assumption and say it's *flat* - that everyone in the US lives until a certain age (say, 75) and then abruptly dies. Then, we can estimate the number of schoolchildren in the country as $3 \cdot 10^8 \cdot \frac{13}{75} \approx 5 \cdot 10^7$.

Ok, but what about the real question? Since we're just trying to get an order-of-magnitude answer here, let's assume that most of the cost of education goes to paying teacher salaries. I remember from my own schooling that a typical class size might be something like 25 students, which means we have $5 \cdot 10^7/25 \approx 2 \cdot 10^6$ teachers to pay. A teacher might make like 50 grand a year in wages, but they also get decent benefits and a retirement plan - the ones who work in public schools, anyway - so let's estimate the total compensation of a teacher as 100,000 dollars per year. That translates into 200 billion dollars a year, just to pay the teachers. So as a rough estimate, let's say that K-12 education is a $\boxed{\text{200 billion dollar}}$ a year industry.

Does that number make sense? Well, the US GDP is about 14 trillion, so this means that we spend a few percent of our national wealth on education. Sounds about right.

What does the real data say? According to the 2007 Statistical Abstract of the United States[1], we really spend 511.2 billion per year on all K-12 education. Of course, it takes more to run a school district than just teachers; you also need administrators, janitors, counselors, secretaries, special needs professionals, etc. And, we neglected to include any non-labor costs like books, food, or maintenance. So we undershot a bit, but still got to within a factor of a few. If you wanted, you could have included a correction of a few hundred billion to account for other costs.

[1] http://www.census.gov/compendia/statab/2007/tables/07s0205.xls

Entertainment

One way to approach this problem is to estimate the revenues of the key members of the entertainment industry and add them up. The problem with doing that is deciding what exactly to include under "entertainment." That could mean a lot of things, and it turns out that the entertainment sector is not dominated by a few big industries like movies or TV. Unless you were very careful, you probably undershot the answer by a factor of ten or so if you used this approach.

How else could I estimate the size? The problem is that people spend money on different things - I'm a movies and music guy myself, but someone else might prefer comic books, or video games, or magazines, or gambling. It's too easy to forget something big, so instead of estimating industry-by-industry, I'll work from my personal budget and then assume that, whatever they're spending it on, other people are spending money on entertainment at about the same rate I am.

Personally, after dropping 60 dollars a month for cable + Netflix, I'd say I spend about 10 dollars a week on movies, music, books, concerts, and so forth. That's a total of 100 dollars a month, or about 1200 dollars a year.

Is that number reasonable? I'll try to estimate it in another way. As you all know, grad students at Berkeley make roughly 1600 dollars per month, about 1200 dollars of which we immediately fork over as rent, food, clothes, and other necessities. That leaves 400 dollars, which means that if I can indeed afford to spend 100 dollars a month on "entertainment" and still save some money. Note that this means I spend about 6 percent of my monthly income on entertainment, which sounds reasonable.

Is that number about right for an average American? Well, I make less money than average, but I also don't have any kids to support. What's more, I don't really have the time to enjoy much more entertainment than I already am. Even if I had more money, I doubt I'd go to movies or concerts more often. So assuming I'm about as busy as your average American, I think it's reasonable to extrapolate from myself here.

So, if everyone in the United States spent money like I do, the entertainment industry would have a revenue of about $3 \cdot 10^8 \cdot 1200 \approx \boxed{\text{400 billion dollars.}}$

Unfortunately, while the Department of Commerce does keep data on US GDP by industry, it doesn't break down the industries in way that would be useful for us here. Instead, I found a website[2] that gives a total value of 923.9 billion in 2008 for the "Entertainment and Media" industry. This figure includes things like advertising on FM radio that I didn't even begin to take into account here, and anyway, a factor of two isn't bad.

Sports

Lots of people calculated the revenue of a typical major spectator sport league (NBA, NFL, etc.) and got an answer of ≈ 5 billion. That's turns out to be correct, but that number only represents a small fraction of the "sports" industry as compared to things like merchandise and advertising. It can be hard to estimate advertising revenue, but a lot of it comes from sporting goods companies selling things like jerseys or shoes. To get a handle on those things, it's again best to figure out how much you personally spend on sports (which could mean sporting goods, recreational activities, etc...), and then extrapolate to the rest of the population. Note that if everyone in the US spent just $1.50 a month on something sports-related (a pair of socks from Nike), then that would be 5 billion a year right there.

[2] http://www.plunkettresearch.com/Industries/EntertainmentMedia/EntertainmentMediaStatistics/tabid/227/Default.aspx

Let's once again assume I am typical here, if not in my particular habits, then at least in my total expenditures. I don't go to very many games, and I don't spend that much on sporting goods, but I do use the RSF. As a student I don't have to pay very much for the privilege, but if I were to buy a similar membership on my own it could easily be several hundred dollars/year[3]. That has to come from somewhere (tuition), so I should count it under my personal expenditures. Let's include another few hundred to account for miscellaneous equipment I might buy and say I spend 1000 dollars a year on sports. Like entertainment, that's also a few percent of my income, which sounds about right.

Not everyone has a gym membership, but other people probably go to games and buy more athletic clothing than I do, so for lack of anything better to do let's assume everyone else spends money on sports about like I do. If so, sports would be a $\$3 \cdot 10^8 \cdot 1000 \approx \boxed{\text{300 billion dollar}}$ industry. An internet search[4] shows the real value to be 441.1

Restaurants

Following the same approach as above, I eat out maybe 4 times a week at an average of 10 bucks per meal. That's 40 dollars a week, 160 a month, and 1920 dollars per year. If everyone in the country spent that much, the restaurant industry would make $3 \cdot 10^8 \cdot 1920 \approx \boxed{\text{600 billion}}$ per year. The real revenue for 2008, according to an industry website[5], was 566 billion.

2 Astronomy Job Market

> *What percentage of people who graduate with Ph.D.s in astrophysics take up faculty positions in one of the top 10 largest (as judged by the size of the school's astronomy budget) research universities in the United States?*

It's probably safe to assume that Berkeley has a fairly typical top-10 astronomy program, or at least that the ratio of students to faculty is about right. Thus, if we look at Berkeley's astrophysics and astronomy Ph.D production and its demand for professors, we have a reasonable estimate of the proportion of Ph.Ds from the top programs hired into the top schools. Now, the question asks about *all* astro Ph.Ds. The top 10 largest programs probably produce a vast majority of quality Ph.Ds, so for now, let's ignore the smaller (budget) institutions. Also, we'll keep in mind that there is probably some small net flux of Ph.Ds into the US from other countries.

A quick look at the Astronomy Department Directory reveals (or maybe you already knew it) that there are about 20 professors and 50 grad students working in Astronomy. But ∼5 of the 50 grad students actually hail from Physics. We should either subtract them off, or account for the full contribution to astro-related faculty and students from Physics. We'll take the easier route of subtracting them off, since they probably amount to increasing the absolute numbers by a factor of 2. So that's 45 graduate students in Astronomy proper.

How many relevant Ph.Ds per year at Berkeley? It takes, on average, 6 years to graduate. So on average there are around $45/6 \approx 7$ new Ph.Ds every year. Professors stick around a long time, and it's reasonable to assume that the department is in a steady state: astronomy is no longer the young, sexy field it once was (was it?). If a professor is hired when he/she is 30, he/she is

[3] According to their website, a membership at the RSF is 744 dollars a year if you aren't a student or faculty

[4] http://www.plunkettresearch.com/Industries/Sports/SportsStatistics/tabid/273/Default.aspx

[5] http://www.restaurant.org/research/

probably likely to stick around until at least 60. On average, then, 2 professors are hired every 3 years, which is in agreement with the recent trend. In 3 years, though, 21 new Ph.Ds are produced. That's a $\boxed{10\%}$ chance that a top-10 astro Ph.D will end up at a top-10 department. Including other sources of Ph.Ds only *lowers* the chances. If including non-top-10 departments doubles the number of Ph.Ds produced, the percentage, obviously, drops to $\boxed{5\%}$.

This is a reasonable figure. According to the American Institute of Physics,[6] there were about 186 Ph.Ds in astro and astron issued per year in 2002 and 2003. There were about 20 new hires at astronomy departments (not including astrophysics), not just limited to the top 10. That's about 10% of the Ph.D production. This may or may not be a typical year, but it's definitely not crazy to think that limiting to the top-10 departments would reduce this by half or more. Remember, a large number of schools probably hire astronomy professors mainly for teaching purposes. In fact, the AIP has a roster of "degree granting" astronomy programs,[7], and it notes there are 75 such programs in the US. Only 37 grant Ph.Ds.

3 Thalassaemia

> *As reported by Judith Tan of Singapore's Straits Times, Dec 7 2005:*
>
> *Thalassaemia is an inherited blood disorder. Those who have thalassaemia-major have a shortage of hemoglobin and need a blood transfusion every month to stay alive. Victims do not live past childhood unless they receive a bone marrow transplant.*
>
> *Those who have thalassaemia-minor carry the gene but do not express it. They can pass the gene to their children but otherwise lead normal lives. The rules for passing the gene are the usual ones: if both parents have thalassaemia-minor, their children have a 25% chance of having thalassaemia-major, a 50% chance of having thalassaemia-minor and a 25% chance of not carrying the gene at all. If only 1 parent has thalassaemia-minor, offspring have a 50% chance of having thalassaemia-minor and 50% chance of not carrying the gene at all.*
>
> *Worldwide, about* 10^5 *children are born each year with thalassaemia-major.*
>
> *What is the probability that you have thalassaemia-minor?*

Let T represent the condition Thalassaemia-major and t represent Thalassaemia-minor. We are told that

$$10^5 \text{ children born e.y. with } T \Rightarrow 4\times 10^5\ t\text{-}t \text{ couples had kids,}$$

neglecting couples with one T partner.

How many babies are born every year? There are about $6{\cdot}10^9$ people, and the population is growing slowly, though in part due to increasing life-spans. To keep the number constant, every person has one child (a couple has two). With an average lifespan of 70 (75 in the US these days, less in much of the world) years, we need a bit less than 10^8 to keep the number of people *constant*. Round up to 10^8 births to account for a little population growth.[8] Assuming a couple can only

[6] http://www.aip.org/statistics/
[7] http://www.aip.org/statistics/trends/reports/astrost.pdf
[8] According to the US Census Website, this is about right.

have one baby per year, and the couples having babies in a given year are a random sample from all couples, this means that the probability $P(t\text{-}t)$ that both members of a couple have t is

$$P(t\text{-}t) \approx \frac{4 \times 10^5}{10^8} = 0.004.$$

Now, assuming couples are random pairs, this means that the probability that an individual has t is

$$P(t) = \sqrt{P(t\text{-}t)} \approx \boxed{6\%}.$$

This proportion should be constant across generations given our assumptions and the rules of genetics.

It's probably not true that all people are at equal risk of having t. In fact according to the U.K. Thalassaemia Society,[9] Thalassaemia "occurs in a line extending through the Mediterranean, the Middle East, the Indian sub-continent and through out South East Asia, in a region including Southern China, Thailand, the Malay Peninsula and many of the islands." If we say this encompasses roughly half of the population of the world, people of the above origin have a probability of about .09 of carrying t, by the same analysis as above, but with only half the total number of births. The rest of the world probably has close to zero chance of carrying t.

4 Bailing Out the Big Three

Justify the size of the bailout package promised to United States automakers in 2008.

Before he left office, Bush agreed to give 17.4 billion[10] in loans to the Big Three car manufacturers Ford, GM, and Chrysler. Let's just try to understand why the the bailout is of the order of 10 billion dollars.

First of all, how many cars do the big three normally sell in a year? Like in the education question, let's do a steady state calculation based on a) the number of cars on the road and b) the average lifetime of a car.

Most families of 4 probably have at least 2 cars so both parents can drive to work, while most single adults need their own cars, so the number of cars per person in the US is probably greater than 0.5 but less that 1.0. Let's say there are 200 million cars in the US, to keep the numbers nice and round[11].

I recall from motor oil commercials that cars last for something on the order of 100,000 miles, if you take good care of them. I know that when I had a car I'd put something on the order of 10,000 miles on it a year, so I'd estimate that cars last $1 \cdot 10^5/1 \cdot 10^4 \approx 10$ years before breaking down. Then, to keep the number of cars steady, about 20 million have to be sold in the US per year. Probably, the number of cars in the US is growing slightly with the population, but let's ignore that effect. Nowadays, something like half of those cars will be American, so let's say the big three normally sell 10 million cars, plus maybe a few more to account for international sales.

If a new car costs 30,000 dollars, then the Big 3 have a revenue of $\approx$ 300 billion per year. However, this year they made less than that because the sales of cars dropped due to the poor economy. We can estimate the decline in sales by assuming it's related to the employment rate,

[9] www.ukts.org/pages/background.htm

[10] http://news.bbc.co.uk/1/hi/business/7791999.stm

[11] According to Wikipedia, the number is actually 250 million

like in the bank bailout problem from lecture. Following that line of argument, the unemployment rate is 7% and then *change* in the rate is 1%, so let's pick an intermediate value of ≈ 5%. This is probably a lower limit in this case, but it's the best I can do without looking up sales figures.

That means that the Big 3 together lost $300 \cdot 10^9 \cdot 0.05 \approx 15$ billion dollars last year. It makes sense that an bailout would have to be roughly equal to the amount the car companies lost last year, so that they afford to keep running long enough to get things turned turned around, or at least until the economy recovers so they can get loans from the banks again. So, it does make sense that the bailout was on the order of $\boxed{\text{10 billion dollars.}}$

Going to wikipedia to verify some of those numbers, I see that Ford[12] and GM[13] have a combined revenue of about 300 billion, and lost close to 40 billion dollars in 2008 (Chrysler isn't publicly traded so it's numbers aren't available, but it's a smaller company than either Ford or GM). Apparently, sales dropped by quite a bit more than 5% (as some of you knew on your problem sets), but it's also true that companies like the Big 3 probably have some cash on hand, so the actual amount they are in the hole is still around 10 billion.

[12] $http://en.wikipedia.org/wiki/Ford_Motor_Company$

[13] $http://en.wikipedia.org/wiki/General_Motors$