Order-of-Magnitude Physics – Problem Set 12

Problem 1. Random Walking Can Be Efficient

Inspired by Howard Berg's "Random Walks in Biology" (Princeton University Press).

Consider a cell of radius $a \approx 5\,\mu\mathrm{m}$. It absorbs a certain desirable molecule at special receptor sites distributed over the cell membrane surface. These receptor sites are patches each having a radius of $s \approx 10\mathring{A}$, and there are N of them.

The desirable molecules float around in the medium in which the cell lives. Their diffusivity in the medium is D. They encounter a receptor site by *pure chance*. The time-averaged concentration of molecules at the surface of the receptor site is zero (because such molecules are quickly and greedily absorbed by the site). Far away from the receptor site, the concentration is some fixed value n.

(a) Assume that the entire cell surface is covered with receptor sites. Estimate the rate R (number per time) at which molecules are absorbed into the cell. Call this rate R_{max}.

(b) Now relax the assumption that the entire cell surface is covered. Estimate the minimum value of N such that $R = R_{\mathrm{max}}/2$.

(c) For the value of N in part (b), what fraction of the cell surface is covered by receptor sites? Can the cell surface accommodate many different kinds of receptor sites, each absorbing a different kind of molecule?

Problem 2. Kettledrums

Kettledrums (orchestral timpani) are large hemispherical bowls over which a membrane made of leather skin is stretched. The diameters of timpani range from 23 to 30 inches, and the frequencies at which they emit range from 60 to 400 Hz. Estimate the strains which characterize kettledrum membranes. Take the elastic modulus of leather skin to be $\mathcal{M} \sim 10^9\,\mathrm{dyne\,cm^{-2}}$.

Problem 3. Piano Strings

To make sound, piano strings are struck impulsively with a hammer 1/7 of the way down their lengths. Whence 1/7?

Problem 4. Freeway Noise

According to the Federal Highway Administration of the US Department of Transportation, a residential home is "impacted by noise" if the noise level exceeds 70 dBA for 10% of the time.

Estimate the distance away from Highway 80 that a home would need to be to just avoid noise impaction. Repeat for a home that experiences four times less noise.

Problem 5. Ask Your Own Question

Ask an OOM question of your own. You don't have to answer it.