Order-of-Magnitude Physics – Problem Set 4

Guidelines:

- You are encouraged to rely as much as possible on your own wits. For example, looking up a "fundamental" parameter such as the mass of hydrogen is fine (and you don't need to cite such a routine task), but looking up advertising revenues is not as interesting or as instructive as trying to estimate it yourself. Credit will be awarded for independence and ingenuity.

Problem 1. Tabletop Dynamos

Magnetic fields $\vec{B}$ in conducting fluids are governed by the induction equation:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \kappa_{\mathrm{M}} \nabla^2 \vec{B} \tag{1}$$

where the first term on the right-hand side governs how magnetic flux is carried along by ("frozen into") fluid parcels, while the second term is the Ohmic dissipation term derived in class. The fluid velocity is $\vec{v}$, time is t, and κ_{M} is the magnetic diffusivity.

Magnetic dynamos are fluid systems whose motions can sustain magnetic fields.

(a) Were fluid motions in the Earth's core to cease, how long would it take the Earth's magnetic field to decay?

(b) Why is it so difficult to study dynamos using tabletop experiments?[1]

Problem 2. Fade to Black

White dwarfs represent collapsed end-states of solar-mass stars that have expired their nuclear fuel. Typically they have masses $M \approx 1 M_{\odot}$ and radii $R \approx 1 R_{\oplus}$. Their interiors are composed of a crystalline lattice of carbon and oxygen nuclei, bathed in a sea of free degenerate electrons (in short, "an ultra-dense metal made of carbon and oxygen.")

Heat in white dwarf interiors is transported by electron conduction. Heat in white dwarf atmospheres is transported by radiation. Take the core temperature to be T_{core} and the atmospheric temperature to equal T_{eff} (effective temperature of a blackbody). When born, white dwarfs have $T_{\mathrm{eff}} \approx 3.5 \times 10^4$ K and $T_{\mathrm{core}} \approx 1 \times 10^7$ K. Assume that $T_{\mathrm{core}}/T_{\mathrm{eff}}$ maintains this ratio of 300 over the entire cooling history of the white dwarf.

[1] Despite the difficulty, the Princeton plasma physics community has been successful.

For this problem, neglect corrections to the specific heat that might arise from $T < T_{\rm Debye}$.[2]

Parts (a) and (b) can be considered simultaneously.

(a) Shapiro & Teukolsky, in their classic textbook "Black Holes, Neutron Stars, and White Dwarfs," state that as white dwarfs cool, almost all of their interiors can be modelled as isothermal. That is, the temperature equals $T_{\rm core}$ nearly everywhere inside the dwarf. However, they offer no quantitative justification. Provide one.

(b) What are the luminosities of the absolute faintest white dwarfs in the universe today?[3] Express in solar luminosities $L_\odot$.

Problem 3. The Earth Wobbles (But Won't Fall Down)

As reported in the August 2006 issue of Physics Today: Using global positioning system (GPS) data, Lambert et al. (2006, Geophys. Res. Lett.) have measured irregular, day-to-day displacements in the Earth's rotational pole position.

Explain why these changes in spin axis orientation occur and predict an amplitude (in units of length) for the displacements.

Problem 4. "If Frogs Can Fly..."

"...then there is no reason why John Major cannot be Prime Minister"—Reaction of the press after researchers at the Nijmegen High Field Magnet Laboratory in the Netherlands announced they had successfully levitated a 1-cm diameter baby frog in their Bitter magnet.

A Bitter magnet is essentially a solenoid where the usual coil of wire is replaced by vertically thin, radially wide plates stacked helically—like a continuous, tightly wound, spiral staircase. You can see pictures of "Bitter plates" in some of the following excellent tutorial slideshows: http://www.magnet.fsu.edu/mediacenter/slideshows/index.html, created by the National High Magnetic Field Lab at Florida State University.

Bitter plates are punched with holes so that water can cool them when current flows through the plates. The bore hole (axis) of the solenoid is left empty for researchers' test subjects. In the case of Andrey Geim's experiments, subjects have included water droplets, hazelnuts, mice, and pizza.[4]

[2] These corrections turn out to be of order unity; if you look at Shapiro & Teukolsky, there are large numerical coefficients that happen to cancel out any Debye corrections that we might naively make. In the same vein, Kittel ("Introduction to Solid-State Physics") points out that the thermal conductivity of metals is the same for $T > T_{\rm Debye}/10$ but does not explain where the "10" comes from; see the class hand-out for the thermal conductivity curve of copper to verify that this is true.

[3] Sometimes referred to as black dwarfs.

[4] In 2000, Andrey Geim won the Ignobel Prize for his levitation experiments, sharing it with Sir

Model the Nijmegen magnet as follows. The plates are composed of copper, have an inner radius of $R_1 \approx 2\,\text{cm}$, an outer radius $R_2 \approx 20\,\text{cm}$, and are stacked such that there are $n \approx 10$ turns per cm. The length of the solenoid is $L \approx 30\,\text{cm}$. The magnet is oriented so that its axis points parallel to Earth's gravity.

(a) Decide where the frog can levitate (top, middle, or bottom of solenoid). Estimate the magnetic field required to levitate the frog. Give both a symbolic answer and a numerical estimate in Teslas.

(b) How much power is dissipated in the electromagnet? State in Watts.

(c) Repeat (a) and (b) but for you instead of the frog. Is human levitation practical?

Problem 5. Ask Your Own Question

Ask an OOM question of your own. You don't have to answer it.

Michael Berry (of Berry phase fame) who provided the underlying theory.