## Astro 149 – Planetary Astrophysics – Problem Set 1

## Due Thursday in class.

Readings: Course reader, chapter on molecular motion; Landstreet, chapter 4, especially sections 4.1–4.3.

Work symbolically as much as you can. Plug in numbers only when you absolutely must.

**Problem 1.** Cooling Molecular Clouds and Star Formation

Consider again the gravitational collapse of a $\sim 1\ M_\odot$ clump of gas and dust of radius $\sim 0.1\,\mathrm{pc}$. Assume the temperature of the cloud is such that the cloud is marginally Jeans-unstable. The gas consists predominantly of molecular hydrogen.

As the cloud collapses inward, its density increases. In the absence of cooling mechanisms, the gas will heat adiabatically. If the temperature rises too quickly compared to the increase in density, the cloud will become Jeans-stable, cease to collapse, and fail to form a star.

Fortunately, efficient cooling mechanisms are present in today's interstellar medium. Most notable among these is cooling by dust grains. Hot gas molecules can collide with dust grains and heat them conductively. The dust grains can then radiate away this energy by thermal emission of infrared radiation (provided the cloud is optically thin to this infrared radiation.)

The cloud can stay Jeans-unstable if the timescale for cooling is shorter than the timescale for gravitational contraction (the latter is the same as the heating timescale). That is, the collapse can proceed nearly isothermally rather than adiabatically.

(a) What is the timescale for gravitational contraction of a cloud of mass density $\rho$? This is the time it takes for the cloud to decrease its radius by about a factor of 2.

A quick way to estimate this time is to ask how long it takes a test particle to orbit *just above* the surface of a uniformly dense body of mass $M$ and radius $R$. You can choose to use this orbit time as the contraction time—you will be off by a factor of a few—or you can do it your own way. (If you want to account for the factor of a few, think about eccentric orbits. A particle on the surface of the spherically collapsing cloud is like a particle on a radially plunging orbit of eccentricity equal to one. Calculate the orbit time of such a maximally eccentric orbit, then divide by two.)

Express your answer in terms of $G$ (the gravitational constant) and $\rho$.

(b) Express symbolically the mean thermal speed of $H_2$ molecules in the cloud, $v$. Set the translational kinetic energy of a hydrogen molecule equal to 3kT/2, where $k$ is

Boltzmann's constant and $T$ is the kinetic temperature of the molecules.

(c) What is the timescale for cooling? This is the time for a hot gas molecule to collide with a dust grain. Assume canonical parameters for the interstellar medium today: dust is in the form of spheres of radius $r = 0.1\,\mu$m, each sphere has an internal mass density $\rho_p \approx 1\,\mathrm{g\,cm^{-3}}$ (this is of order the density of ordinary, uncompressed bulk matter in the universe), and the volumetric mass density (in space) of dust is $\rho_d \approx Z\rho$, where $Z \approx 10^{-2}$.

Assume the velocity of dust grains is small compared to the velocity of hydrogen molecules.

Express your answer symbolically in terms of $r$, $\rho_d$, $T$ (the temperature of the gas molecules), $\mu_g$ (the mass of a gas molecule), $\rho_p$ (the internal mass density of a dust grain), and $k$ (Boltzmann's constant).

(d) Initially, the cloud is marginally Jeans-unstable; that is, its mass is just slightly above the Jeans mass. At this time, which is shorter, the cooling time or the contraction (heating) time?

Note that I have not given you a temperature $T$. Use the fact that the cloud is marginally Jeans-unstable to solve for $T$.

(e) Explain why you might or might not expect to maintain the inequality of timescales found in (d) as the cloud collapses. Is this calculation promising for the formation of stars?

**Problem 2.** Lipstick

Estimate the mass in lipstick that Brittney Spears has ingested over her life so far. Express in whatever units seem most meaningful to you.