Astro 162 – Planetary Astrophysics – Problem Set 10

Due Thursday in class.

Readings: Landstreet sections 5.1–5.4. All of section 7.5. Finally, last few pages of the reader on the K-T catastrophe.

Problem 1. Dogs and Cats Living Together

Estimate the size of the meteorite that crashed into the Earth 65 million years ago to form the Chicxulub crater and to wipe out life as the dinosaurs knew it. We saw in class that the crater was about 180 km in diameter.

Problem 2. J_2

A body's J_2 has simple physical interpretations. Its order of magnitude can be estimated as either

$$J_2 \sim \frac{R_{\rm equator} - R_{\rm pole}}{R_{\rm equator}} \qquad (1)$$

where $R_{\rm equator}$ is the body's equatorial radius (measured perpendicular to the spin axis) and $R_{\rm pole}$ is the body's polar radius (measured parallel to its spin axis). Alternatively, we can estimate it as

$$J_2 \sim \frac{\text{Rotational Kinetic Energy}}{\text{Gravitational Potential Energy}}\,. \qquad (2)$$

(a) Use (2) to derive an analytic expression for J_2 in terms of the spin frequency of the planet (Ω), the mean density of the planet (ρ), and any natural constants.

(b) Estimate J_2 for the Earth and for Saturn without looking up the answers directly. Then compare your answers to the truth by looking up the answers directly.

Problem 3. Weirdness on the Sphere

a) Long-period comets, unlike their short-period brethren, come from all directions on the sky. We say that their orbital planes are distributed isotropically; i.e., the orbit pole vector (the vector perpendicular to the plane of a comet's orbit) can point anywhere on the celestial sphere with equal probability.

Define the orbital inclination, i, to be the angle between the orbital plane of a comet and some fixed reference plane. The inclination can take any value between $i = 0$ and

$i = 180°$. "Prograde" orbits have $0 \leq i < 90°$, "retrograde" orbits have $90° < i \leq 180°$, and "polar" orbits have $i = 90°$.

Derive the differential inclination distribution, dN/di, for isotropically oriented cometary orbits, where dN is the differential number of comets having inclinations between i and $i + di$. Normalize your distribution so that $\int_0^{180°} (dN/di) di = 1$.

(b) Given your answer in (a), are nearly co-planar ($i \approx 0$) orbits more common, less common, or just as common to find as nearly polar ($i \approx 90°$) orbits? (Are you weirded out by your answer?)

(c) Geoff Marcy and his collaborators measure the line-of-sight Doppler shifts of stars to infer the presence of planetary companions around those stars. The extrasolar planet tugs gravitationally on its parent star and induces the star to revolve about the common center of mass. A key unknown in the measurement is the angle between the orbital plane of the planet and the plane of the sky (the plane of the sky is perpendicular to the observer's line of sight). Call this angle i. If $i = 0$, the orbit plane coincides with the sky plane and there will be no line-of-sight Doppler shift measurable. The maximum line-of-sight Doppler shift arises for $i = 90°$. Here, $0 \leq i \leq 90°$ only.

For a given measured Doppler shift, Marcy et al. cannot measure the mass of the planet, m, directly. They can only measure $m \sin i$ because of the unknown orientation of the planet's orbital plane. For example, if Marcy et al. measure an $m \sin i = 1$ Jupiter mass, they don't know whether it's a 1 Jupiter mass planet whose orbit is oriented perpendicular to the sky plane ($i = 90°$), or whether it's a 2 Jupiter mass planet for which $i = 30°$. (And in the ridiculously extreme limit, they don't even know if it's an infinite mass planet with $i = 0°$!)

Fischer et al. measured the Doppler wobble of the star HD217107 and derive $m \sin i = 1.23 M_J$, where M_J is a Jupiter mass. Calculate the probability that the unseen companion is not a planet but a star whose mass exceeds 80 Jupiter masses.

Problem 4. The Nougat in the Center: Isothermal Extreme

Repeat Problem 4 of last week's problem set #9, but assume instead that the gas behaves *isothermally* when it accretes onto the rocky core, not adiabatically as was assumed last week. That is, assume this time that $T_s = T_0$.

(a) Find a new analytic expression for ρ_s, using the same procedure as described in last week's problem set.

(b) Solve for a new critical radius of the core, R_c, such that $M_{\rm env} = M$. Give an analytic expression for R_c in terms of the variables given. Also give a numerical estimate for the critical mass M_c in terms of $M_\oplus$.

For actual numbers, see last week's problem set.

In general, isothermal and adiabatic conditions represent two extremes. The isothermal condition assumes that the gas is able to cool perfectly (say, by radiation to space) back down to its original temperature even when it undergoes compression as it accretes onto the core. The adiabatic condition says that the gas is totally unable to cool and therefore heats up (as if it were wrapped in insulating styrofoam) when it undergoes compression as it accretes onto the core. The truth lies somewhere in between the isothermal assumption and the adiabatic assumption. The true critical core mass (5–10 $M_\oplus$) should lie in between the answer you get this week and last week's answer.