

Astro 162 – Planetary Astrophysics – Problem Set 11

Due Thursday in class.

Readings: None.

Problem 1. Tearing Up the Oort Cloud

(a) Consider an invading star of mass $M_* = 0.5M_\odot$ speeding through the Oort Cloud at a velocity relative to the Sun of $V_* = 30$ km/s. Idealize the Oort Cloud as a shell of comets at heliocentric distance $R = 10^5$ AU. Estimate how close the invading star needs to come near a comet to gravitationally unbind that comet from the Oort Cloud. Call this maximum approach distance d (so for all impact parameters less than d , the comet is likely to get thrown out of the Cloud). You may use the impulse approximation as described in class, but only if you justify its use.

(b) Over the age of the Solar System (4×10^9 yr), what fraction of Oort Cloud comets at $R = 10^5$ AU are likely to be ejected into interstellar space by such single, violent scatterings? Take the mean stellar number density to be $n_* \sim 0.1$ stars/pc³, the mean stellar mass to be M_* as above, and the stellar velocity dispersion to be V_* as above.

(c) Repeat (b), but for $R = 10^4$ AU.

Problem 2. Bye-Bye Bopp, So Long SOHO

(a) Take Comet Hale-Bopp to be a sphere of dirtyish water ice of radius $R \sim 10$ km and albedo $A \sim 0.1$. Water ice that is naked before the Sun tends to sublimate at heliocentric distances inside about 3 AU.

Hale-Bopp's perihelion distance is $q = 0.91$ AU and its aphelion distance is $Q = 372$ AU. ESTIMATE the number of times, N , that Hale-Bopp will loop around the sun before it evaporates, assuming the comet keeps the same orbit it has today.

(b) SOHO (SOlar and Heliospheric Observatory) is a satellite that stares at the Sun 24 hours a day, relaying ultraviolet images of the Sun back to Earth every 30 minutes. SOHO lives at the second Lagrange point (L2) between the Earth and the Sun.

An unexpected side benefit of SOHO was that it discovered thousands of sun-grazing comets. These comets had perihelia ranging from $q \sim 1$ –10 solar radii. In fact, SOHO is history's most successful comet-hunter, having discovered 1200+ comets in 5+ years. An image of a sun-grazer can be found at

http://umbra.nascom.nasa.gov/comets/twin_comets_19980601.gif

SOHO sees these comets going towards the sun, but it doesn't see them coming away. Given this observation, estimate the maximum radius that these sun-grazers could have.

Problem 3. Bubble Bubble, Toil and Trouble

The pressure inside a bubble differs slightly from the pressure outside. Use the Buckingham Pi Theorem (otherwise known as dimensional analysis) to write down an analytic expression for this pressure difference.

Salient properties of liquid bubbles:

- The surface consists of a liquid.
- The interior contains a gas.
- The bubble may be embedded in a gas or liquid.

Examples:

- Soap bubble of air in air (surface is made of liquid soap).
- Champagne bubbles (surface is made of liquid champagne; interior is largely carbon dioxide).
- Coke bubbles (surface is made of liquid Coke; interior is largely carbon dioxide).
- Bubbles exhaled from scuba diver (surface is made of water; interior is largely carbon dioxide and nitrogen).

Ask whether the pressure inside the bubble is greater than or less than the pressure outside.

Your answer should suggest why it is that tiny bubbles in Coke cans make a huge racket.