Astro 162 – Planetary Astrophysics – Problem Set 12

Due Thursday in class.

Readings: Section 1.5 of Landstreet

Problem 1. Soup Bowls and Ocean Basins

A flat bottomed bowl of radius R is filled with water to depth H.

(a) Derive an approximate analytic formula for the period P of the sloshing mode.

(b) Use your formula to estimate P for a soup bowl and for the Earth's ocean basin.

Problem 2. The Storm That Launched a Thousand Waves

Following a winter storm above the ocean, the interval between waves at California beaches declined from 17–19 s on Sunday, to 16–18s on Monday, and to 15–16s on Tuesday. Typical values are 10–11 s.

These waves are *surface gravity waves on deep water* (or somewhat misleadingly, "deep water waves.")

(a) What was the maximum sustained wind speed during the storm?

(b) How distant was the storm from the beaches?

(c) How long ago did the storm take place?

(d) What are upper limits on the size and duration of the storm?

Problem 3. Tidal Disruption and the Roche Zone

This problem examines why ring systems about all the giant planets occupy planetocentric distances that are less than $\sim$2 planetary radii.

(a) Consider a perfectly rigid, spherical satellite of radius R_s, mass m_s, and density ρ_s orbiting a planet of radius R_p, mass m_p, and density ρ_p. Assume the satellite to be in synchronous lock; this means its spin period matches its orbital period. Take the satellite's orbital semi-major axis to be a_s and its orbital eccentricity to be zero.

A marble rests on the surface of this spinning satellite. The spin of the satellite tries to spin it off. The tidal field of the planet also tries to pull it off. The only force trying to keep it glued to the satellite is the satellite's own gravity. For small enough a_s, the marble will fly off. What is this minimum semi-major axis, $a_{s,1}$? Express in terms of ρ_s, ρ_p, and R_p.

First decide at which location on the satellite the marble is most unstable. Is it on the pole of the planet or on the equator? Is it right between the planet and the satellite? On the far side of the satellite away from the planet? At right angles with the center of the satellite and the planet? Once you decide the most unstable location, write down force balance for the marble at that location.

(b) Now consider a marble floating on a perfectly strengthless, fluid, synchronously rotating satellite. The satellite's shape is now free to distort because the satellite is sitting in the tidal field of the planet and because the satellite is spinning.

ESTIMATE the semi-major axis of the satellite, $a_{s,2}$, inside of which the marble flies off the watery satellite. This is a repeat of (a) except that you will need to account for the distorted shape of the satellite; the satellite has an enhanced size due not only to the TIDE raised on it by the planet, but also due to its SPIN.

You should at least decide whether $a_{s,2}$ should be larger or smaller than $a_{s,1}$.

Bring to bear all your powers of estimation. Credit will be awarded for precision and elegance.

(c) At orbital semi-major axes of less than $\sim$2 planetary radii, there are no satellites whose sizes exceed 100 km, but there are rings composed of meter-sized boulders and smaller debris, and 10 km-sized satellites. Given your answers in (a) and (b), explain why these observations make sense.