# Astro 162 – Planetary Astrophysics – Problem Set 3

## Due Thursday in class.

Readings: Landstreet section 5.9; also pages 139-141 on radioactive dating; skim section 5.6

**Problem 1.** Disk Heaven

Consider once again the minimum-mass solar nebula, a circumstellar disk of gas and dust of solar composition orbiting the young sun.

X-ray satellite observations of young stars reveal that they are much more luminous in X-rays than the present-day sun. Some of these X-rays will strike the circumstellar disk and ionize its outermost layers of gas.

Assume this ionized outer layer of hydrogen is *everywhere* at a temperature of $T = 10^4\,\mathrm{K}$. (One could compute its temperature from first principles by equating the heating rate due to photoionization to the cooling rate due to radiative de-excitation of collisionally excited ions.)

(a) What is the thermal speed of ions in this hydrogen plasma? Express in [km/s].

(b) At what critical disk radius, $r_{crit}$, does the vertical scale height, $h$, of this $T = 10^4\,\mathrm{K}$ plasma equal the disk radius? In other words, at what critical disk radius does the aspect ratio, $h/r$, of the plasma disk equal 1? Answer symbolically in terms of $G$, $M_\odot$, $k$, $T$, and $\mu$ (the usual symbols; $\mu$ is the mass of the hydrogen ion), and numerically in [AU]. Remember that $h$ is determined by hydrostatic balance in the vertical direction; we derived an expression for the vertical scale height in class, which you may either use or derive yourself.

(c) Imagine that you are a parcel of plasma at disk radius $r = r_{crit}$ and hovering above the disk midplane at the full scale height $z = h$. Calculate the escape speed from the sun at your position, and compare to your thermal speed. Would you expect to remain gravitationally bound to the sun?

(d) Repeat (c) but at $r \gg r_{crit}$ to explain quantitatively why plasma might or might not remain bound to the sun at those great distances. Repeat also for $r \ll r_{crit}$.

(e) Comment on the relevance of your answers above to the metallicity gradient with heliocentric distance in the giant planets. In other words, can you understand why Jupiter at 5 AU is closest to solar composition but Neptune at 30 AU is heavily enriched with metals?

**Problem 2.** Terrestrial Rain Fall

(a) Estimate the mean annual rain fall [in cm] that would result if all the solar radiation intercepted by the earth went into evaporating water.

(b) List 2 factors that would reduce your estimate in (a).

**Problem 3.** Forming Terra

Is gravitational focussing required to form the Earth?

Give a yes or no answer and justify it quantitatively. Credit awarded for care and precision in estimation.

**Problem 4.** Live Aluminum-26 in the Ancient Solar System

In 1976, Lee et al. (1976, Geophys. Rev. Letters) published the results of isotope analyses of Aluminum (Al) and Magnesium (Mg) in a calcium-aluminum inclusion (CAI) in the Allende chondritic meteorite. Different minerals in the CAI (fassaite, melilite, anorthite-B, anorthite-G) gave different isotope ratios. But together these ratios formed a trend: higher abundances of $^{26}$Mg were found with higher abundances of $^{27}$Al. The data looked something like this:

| MINERAL | $^{27}$Al/$^{24}$Mg | $^{26}$Mg/$^{24}$Mg |
|---|---|---|
| Fassaite | 0 | 0.1400 |
| Melilite | 10 | 0.1405 |
| Anorthite-B | 133 | 0.1467 |
| Anorthite-G | 240 | 0.1520 |

Here $^{27}$Al/$^{24}$Mg is the number abundance of $^{27}$Al atoms relative to the number abundance of $^{24}$Mg atoms. The same notation applies to $^{26}$Mg/$^{24}$Mg. You can verify that these 4 points make a pretty good line.

Now it is known that $^{26}$Al is an unstable isotope of Aluminum that beta-decays with a half-life of a mere $0.7 \times 10^6$ yr into $^{26}$Mg. Thus, some of the $^{26}$Mg found in the CAI may be radiogenic (it came from $^{26}$Al). By contrast, $^{24}$Mg is non-radiogenic and is used here as a reference. Furthermore, $^{27}$Al is completely stable.

Use arguments similar to those used in U-Pb dating (as discussed in class), and the data above, to estimate the $^{26}$Al/$^{27}$Al abundance ratio at the time the CAI solidified. (This problem does not demand any fanciness; please don't think too complicatedly.)

We may see later that live $^{26}$Al was a major heat source in ancient planetesimals, able to melt their interiors.