

## Astro 162 – Planetary Astrophysics – Problem Set 4

Due Thursday in class.

Readings: Landstreet all of Chapter 6 pages 145–166 (skip everything starting with “Internal Heat Sources”). Just skim pages 154–159 on “Chemical and Mineral Composition” but be sure to appreciate Figure 6.12.

### Problem 1. Rounding Rocks

Estimate, to order-of-magnitude and from first principles, the critical size for rocks in space above which they would be “nearly spherical.” As with all ill-posed but potentially interesting questions, it is your responsibility to re-pose it well: define quantitatively yourself what you would consider “nearly spherical.”

Check your answer against what you know about asteroids (from, say, the reading).

### Problem 2. Collisional Cascades

Consider an ensemble of bodies having a differential size distribution that takes the form of a power law:

$$\frac{dN}{dR} \propto R^{-q}. \quad (1)$$

Here  $dN$  is the differential number of objects having radii between  $R$  and  $R + dR$ , and  $q$  is a constant. Note the minus sign in the above equation, and the fact that the equation is a proportionality (not an equality!)

It is known that for these bodies, the yield stress (having units of energy per volume, or equivalently force per area) is

$$Y \propto R^s \quad (2)$$

where  $s$  is another numerical constant. For this problem, interpret the yield stress  $Y$  as the stress required to catastrophically disrupt a target of radius of  $R$ .

(a) Repeat Dohnanyi’s derivation of a catastrophic collisional cascade in quasi-steady-state to derive  $q$  as a function of  $s$ . Use the same assumption of Dohnanyi’s, that the relative velocity between colliding bodies is constant.

As a check on your answer, verify that you obtain  $q = 7/2$  for  $s = 0$ .

(b) For bodies that are held together by self-gravity rather than by chemical bonds, what is  $s$ ? Thereby infer  $q$  for such “rubble piles.”

### Problem 3. Spin City

Examine Figure 1 of the Course Reader from Hikida and Mizutani on “Tensile Strengths of Asteroids Predicted from the Relation of Asteroid Size and Spin Period.”

Invent a quantitative, order-of-magnitude theory for the apparent rotation period limit of  $\sim 2$  hours for asteroids larger than 1 km. That is, explain quantitatively why no asteroid larger than 1 km is observed to rotate with a period faster than 2 hours.

Be sure to explain how your theory accounts for the fact that pebbles (of the size that you find on the ground, here on Earth) can certainly rotate with periods much faster than 2 hours.

Explain carefully your reasoning and assumptions. You don’t need to write an essay, but just writing down equations without any explanation or commentary will not suffice.

### Problem 4. How We Know What We Know About the Kuiper Belt

Objects orbiting the Sun both reflect and absorb sunlight. The fraction of sunlight that a body reflects is given by its albedo,  $A$ . The rest is absorbed and heats the object. The absorbed energy is emitted as thermal radiation.

Kuiper belt objects (KBOs), orbiting at distances beyond the outermost planet Neptune, are so distant from the Earth that resolved images of them are rarely possible. Most of them are, to us, point sources in both reflected and emitted light.

(a) An astronomer wishes to measure the reflected flux from a KBO. At what wavelengths should the astronomer work, and why?

(b) Astronomer X succeeds in measuring the *reflected* flux. Call this reflected flux,  $F_R$ . This flux is the amount of energy from the object striking a unit area of a detector at Earth per unit time. Assume the astronomer measures  $F_R$  when the object is at opposition—i.e., the Sun, Earth, and the KBO lie along a single line.

Write down a symbolic expression for  $F_R$  in terms of the distance to the object ( $d$ ), the luminosity of the sun ( $L_\odot$ ), the radius of the assumed spherical object ( $R$ ), and the albedo ( $A$ ).

Hint: *reflected* is italicized for a reason. If  $d \gg 1$  AU (as it is for KBOs), how does  $F_R$  scale with  $d$ ? The answer is not the usual inverse square law.

(c) Astronomer Y measures the emitted flux of the same object. The emitted flux is the

re-radiated flux due to absorbed sunlight. Call this emitted flux,  $F_E$ .

Repeat (b), but for  $F_E$ .

(d) At opposition, the KBO is observed to move relative to the distant (fixed) background stars with a proper motion of  $\mu = 3$  arcseconds/hr. This proper motion is due purely to parallax. Estimate  $d$  based on this information, and express in AUs.

(e) Given your answers for (c) and (d), at what wavelengths should astronomer Y be observing? If you were unable to estimate  $d$ , assume  $d = 40$  AU.

(f) Assume astronomers X and Y both know  $d$ . Can astronomer X, working alone, figure out  $R$ ? Can astronomer Y, working alone, figure out  $R$ ? Can astronomers X and Y, working together, figure out  $R$ ?

Explain your reasoning for all parts.