

## Astro 162 – Planetary Astrophysics – Problem Set 5

Readings: Landstreet, Chapter 10, pages 265–274, also page 290

### Problem 1. Deep Impact

Estimate, to order-of-magnitude, the critical size of the meteor plunging into the Earth's atmosphere that would fail to be slowed down significantly by air. That is, rocks of this size would strike the ground at nearly the full initial velocity of about  $\sim 30 \text{ km s}^{-1}$  (of order the orbital speed of the earth around the sun). Assume the rock is made of solid, competent iron and does not shatter in the atmosphere.

The intrepid can STOP READING HERE and try to do this on their own. The more cautious can work through the following steps:

(a) Imagine the asteroid plunging through the atmosphere. The atmospheric density decreases exponentially with height above the ground. The atmospheric density at sea level and the atmospheric scale height are given in Purcell's cheat sheet. Let's say the rock is initially located at a height  $z$  above the ground of 10 atmospheric scale heights,  $z_0 = 10h$ , and is headed straight down with an incoming velocity of  $|v_0| = 30 \text{ km/s}$ .

Write down an equation governing the deceleration,  $dv/dt$ , of the rock. Use what you think is the appropriate drag law. Your equation should just read,  $dv/dt = [TERM]$ , where  $[TERM]$  is a single term that depends only on the density of air (which depends on  $z$ ), the radius of the rock, the internal density of the rock, the velocity  $v$ , and some dimensionless quantities of order unity. I would leave all variables in symbolic form and plug in numbers only at the very end.

Neglect the extra acceleration due to gravity. This is safe to do for the problem of interest, because in the time that the asteroid takes to fall from  $z = 10h$  to  $z = 0$ , gravity only adds to the rock's already frightening velocity by a tiny fraction. You can check this easily, if you wish.

(b) Convert your equation to a linear, first-order differential equation. That is, you should have an equation which reads something like  $dy/dx = f(x)y$ , where  $f(x)$  is some function of  $x$ . (Hint: begin by noting that  $dt = dz/v$ . If you have the correct drag law, note that you should substitute  $y = v^2$ .)

(c) Solve the equation, and impose the boundary conditions to solve for any constants of integration. You can use the math hand-out in class if you wish. You should have an explicit equation for  $v(z)$  (velocity as a function of height).

(d) Calculate the critical radius of the iron meteorite for which  $v(z = 0) = 0.9v(z_0)$ , i.e., the size of the meteor that would only have its initial velocity reduced by 10%. Do bigger things hit the ground faster or do smaller things hit faster according to your solution?

Express your answer in [m].

(e) For bonus points, give a physical interpretation of this critical radius as expressed symbolically. Hint: think column density (equivalent to surface density).

**Problem 2.** Your Mileage May Vary

Using what you know about aerodynamic drag and the energy content of gasoline, estimate the mileage rating of a typical suburban sedan driving at highway speeds. Express in miles per gallon of gas. Explain your reasoning throughout. Give a symbolic expression in terms of whatever variables you need, and a numerical estimate.

**Problem 3.** Hot Tiles

Using what you know about aerodynamic drag (and other physics), estimate the peak temperature of the tiles on the bottom of the Space Shuttle during re-entry. Give a symbolic expression in terms of whatever variables you need, and a numerical estimate.

**Problem 4.** When the Dust Settles

Imagine a dust particle having a radius  $r = 1 \mu\text{m}$  located initially one gas scale height,  $z = h$ , above the midplane of the minimum-mass solar nebula at a heliocentric radius of  $a = 1 \text{ AU}$ . The solar nebula has the full complement of hydrogen gas of surface density  $\Sigma \sim 1.5 \times 10^3 \text{ g/cm}^2$ . In the absence of turbulence, the dust particle will fall vertically through the gas due to the vertical component of the star's gravity (we dealt with this vertical component in class). Assume the dust particle does not move radially as it falls (this is a fine approximation for small dust particles).

Estimate, to order-of-magnitude, how long it takes for this dust grain to fall vertically to the midplane ( $z = 0$ ). Give a symbolic answer in terms of whatever variables you need, and a numerical estimate in [yr]. In particular, specify how does the settling time scales with  $r$  and with  $a$ , for  $a > 1 \text{ AU}$ .

The settling time you have computed is relevant for interpreting the spectra of disks. Old disks in which the dust has settled are less luminous than young disks in which the dust has not settled, because settled (flat) disks absorb less central starlight than unsettled (flared) disks.