Astro 162 – Planetary Astrophysics – Problem Set 6

Due Thursday in class.

Readings: Course Reader, just skim the entire chapter on the Venusian atmosphere by Don Hunten.

Problem 1. Photon Pinball in Clouds

This problem shows how the usual $e^{-\tau}$ attenuation factor for light through an absorbing medium is incorrect for light passing through a purely scattering, 1-dimensional medium. A cloud can be approximated as a purely scattering medium.

Idealize the cloud as a uniform, 1-dimensional slab comprising particles that can only SCATTER light. That is, a photon passing through a cloud gets bounced like a pinball from cloud droplet to cloud droplet, preserving its frequency and never getting absorbed by any droplet. A few photon/pinballs are lucky enough to make it through the cloud, while most get pinballed back out the way they came. We will calculate the fraction that make it through.

Take the cloud to have a droplet density [droplets per cubic volume] η, the droplet radius to be R, and the vertical thickness of the cloud to be z_{max}. Measure vertical distance through the cloud by z, where the top of the cloud is located at $z = 0$ and the base of the cloud is located at $z = z_{max}$.

(a) Write down the optical depth of the cloud, τ.

(b) *Incident* photons from the sun strike the top of the cloud. The photons have a number flux, F_i [number per time per area]. What is the *number density of incident photons* at the top of the cloud? Call this photon number density n_i. These incident photons have NOT been scattered yet by any droplet. Remember that flux is a number density multiplied by a speed.

(c) These photons pinball/random walk through the scattering droplets. Random walks are described by the diffusion equation,

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} \qquad (1)$$

where D is the diffusivity (diffusion coefficient) and n is the photon number density.

Express D in terms of symbols defined above and whatever fundamental constants you deem appropriate.

(d) In steady-state, $\partial n/\partial t = 0$ (the number density of photons everywhere in the cloud does not change with time). Write down the solution to the diffusion equation for $n(z)$. You should have two, as yet unknown, constants of integration.

(e) To solve for the two constants of integration, you need two boundary conditions. The first condition is that $n_t = n(z = z_{max})$. Here n_t is the number density of photons at the base of the cloud. These photons comprise the *transmitted* flux.

The second condition is that the (net, number) flux, F, of photons at $z = 0$ equals the incident flux, F_i (directed down into the cloud) MINUS the outgoing, *reflected* flux, F_r (directed up, away from the cloud into space). Recall Fick's law, which is just another way of writing the diffusion equation, that the (net) flux $F = -D\partial n/\partial z$. So we have $F(z = 0) = F_i - F_r$.

Use the above, and the fact that the incident flux, F_i, must equal the reflected flux, F_r, PLUS the transmitted flux, F_t, to calculate T, the ratio of the transmitted flux to the incident flux, in terms of τ. Are you glad that clouds scatter but do not absorb light?

(f) Evaluate T for the clouds of sulfuric acid on Venus. You may use Hunten's chapter to find numbers for η, R, and z_{max}.

Despite the smallness of T, it is sufficient to drive a strong greenhouse effect on Venus.

Problem 2. *DIFFUSION*: A New Fragrance by 162

Someone opens a bottle of unusually strong perfume in a classroom.

(a) If the perfume molecules travel only by random walking (diffusion), how long does it take for the entire room to smell of perfume?

(b) Does your answer in (a) accord with your experience? If not, why not?

Problem 3. Melting Vesta and Cooking Turkeys

Vesta is an asteroid whose interior may have differentiated chemically, judging from the light basaltic crust that we know to be present on the surface of Vesta. To have differentiated, Vesta must have been hot enough to melt a substantial fraction of its interior.

Assume Vesta was assembled very early in the solar system's history—so early that it contained the full complement of live radioactive Aluminum-26 inferred to be present from primitive meteorites. For those of you who worked out a problem from an earlier set, you know that the full complement means a ^{26}Al/^{27}Al number abundance ratio of 5×10^{-5} (^{27}Al is the common, non-radioactive isotope of Al.) The radioactive decay of each ^{26}Al nucleus into ^{26}Mg releases a gamma-ray photon having 1.8 MeV (mega-electron-volts). The exponential decay timescale is short, 7×10^5 yr, compared to the age of the solar system.

(a) Assume for this part that all of the energy released by the radioactive decay of ^{26}Al is trapped in Vesta. Estimate the fraction of Vesta that melts due to this short-lived radionuclide. Is ^{26}Al promising as a heat source?

You will need to make a reasonable model for the relevant relative elemental abundances in Vesta. You may use the table of solar abundances by Grevesse and Sauval, but remember that while the *relative metal* abundances of Vesta may be close to those of primitive meteorites and therefore close to those of the Sun, Vesta is very much *unlike* the Sun in lacking H and He—it's a rock, not a ball of gas.

(b) Justify quantitatively the assumption in (a) that most of the energy released by ^{26}Al is trapped in Vesta and that little of it gets radiated away into space.

Hint: In your experience, how long does it take to cook a turkey? Apply a scaling argument from turkeys to Vesta, using what you know about by diffusion. Turkeys transport heat by the jostling of molecules from their (oven-air-heated) hot exterior to their cold interior. Rocky celestial bodies do the same, but in reverse: they transport heat from their (radioactively) hot interior to their cold exterior by the jostling of molecules. In both cases, the jostling is random and therefore energy is transported by a random walk. Assume that the heat diffusivity of a turkey is the same as that of Vesta; in fact, all non-metals (insulators) have about the same heat diffusivity, as we will discuss later in class.