## Astro 162 – Planetary Astrophysics – Problem Set 7

## Due Thursday in class.

Readings: Landstreet Chapter 10, including 10.10, but skip sections on "Winds" and "Effects of Planetary Rotation" on pages 275–278; skip section 10.7, and skip the section on "The Vanishing Atmosphere of Mars" on page 288

**Problem 1.** Air Conditioners, Airplanes, and Atmospheric Stability (problem by Goldreich)

Airplanes cruise at altitudes of 10 km above sea level. The outside air at this altitude is at a pressure $p \approx 264$ millibars, and at a temperature $T \approx 223$ K.

(a) Would the air outside the airplane, if adiabatically compressed, be suitable for use in a plane's passenger cabin? Take the ratio of specific heats of air to be $\gamma = c_p/c_v = 7/5$, appropriate for classically excited, rigid diatomic rotating molecules.

(b) Based only on your answer in (a), would you conclude that the Earth's atmosphere below 10 km altitude is stable or unstable to convection? In other words, is the actual temperature gradient in the lower atmosphere sub-adiabatic or super-adiabatic?

**Problem 2.** Just How Tiny is the Tiny Superadiabatic Temperature Gradient?

It is often stated that convection is so efficient at transporting heat that the actual temperature gradient in a convective atmosphere is only slightly steeper than the adiabatic temperature gradient. Here we estimate quantitatively what "slightly steeper" means.

Recall that the Brunt-Vaisala (B-V) frequency is the frequency of buoyant vertical motions in an atmosphere, and is given by

$$\omega_{B-V}^2 = \left[ \left.\frac{\partial T}{\partial z}\right|_{actual} - \left.\frac{\partial T}{\partial z}\right|_{adiabatic} \right] \frac{g}{T}\,, \qquad (1)$$

where $g > 0$ is the gravitational acceleration, $T$ is temperature, and $z$ is height above the base of the atmosphere. Define

$$\Delta\nabla T = \left[ \left.\frac{\partial T}{\partial z}\right|_{actual} - \left.\frac{\partial T}{\partial z}\right|_{adiabatic} \right] \qquad (2)$$

to be the difference between the actual temperature gradient and the adiabatic temperature gradient. We will estimate $\Delta\nabla T$, and compare it to $\nabla T|_{actual}$. Remember that if $\Delta\nabla T < 0$, then $\omega_{B-V}^2 < 0$—in other words, the B-V frequency is imaginary, any vertical motions are unstable, and convection ensues. Since $\nabla T < 0$, $\Delta \nabla T < 0$ means the *absolute value* of the actual temperature gradient, $|\nabla T|_{actual}$, *exceeds* the *absolute value* of the adiabatic temperature gradient, $|\nabla T|_{adiabatic}$; we say the actual temperature gradient is *superadiabatic* (but not by much) in convective atmospheres.

(a) Consider a parcel of gas moving *adiabatically* upwards through a convective (superadiabatic) atmosphere. The parcel has mass density $\rho$ and specific heat [erg/(gram K)] at constant pressure $c_p$. It maintains pressure equilibrium with its surroundings: as it rises, the parcel's pressure matches exactly the surrounding atmospheric pressure (which is decreasing with increasing height). The parcel's temperature decreases adiabatically, while the atmosphere's temperature drops superadiabatically. In other words, the adiabatic drop in the parcel's temperature is not as much as the drop in the surrounding environment's temperature, because the actual temperature gradient of the environment is superadiabatic.

After the parcel has risen length $l$, where $l$ is small compared to the pressure scale height, how much *excess* energy density [erg/cm$^3$] does the parcel carry relative to its surroundings? Use the variables given above. Call this extra energy density $\epsilon$.

(b) Give an approximate symbolic expression for the upward velocity, $v$, of the buoyant parcel after it has travelled distance $l$. Remember that the parcel is unstably buoyant; it experiences an upward acceleration, $(\delta\rho/\rho)g$, where $g$ is the local (downward) planetary gravitational acceleration. You should first understand why $\delta\rho$, the density difference between the parcel and its surroundings, is negative. Reduce your expression to one that DOES NOT contain $\rho$ or $\delta\rho$, but DOES contain $T$.

(c) Assume that convection dominates heat transport through the atmosphere. The atmosphere must transport the full flux of energy that is deposited at its base. Call this flux, $F$ [erg cm$^{-2}$ s$^{-1}$]. Therefore $F = \epsilon v$ (the density of any quantity times the speed with which that quantity moves gives you a flux).

Use $F = \epsilon v$ and (a) and (b) to solve for an approximate symbolic expression for $\Delta \nabla T$. Your answer should depend on $F$, $l$, $T$, $g$, $c_p$, and $\rho$.

(d) Calculate $|(\Delta \nabla T)/(\nabla T)_{actual}|$ for conditions appropriate to Jupiter's atmosphere at a pressure of 1 bar. For this numerical evaluation, the only quantity which is not given by data is $l$, the distance a parcel travels before it dissolves away into its surroundings. No one knows what $l$—the infamous "mixing length" of convection—is. It is reasonable that $l < h$, the pressure scale height of the atmosphere. Here let's charge forth boldly and use $l \sim h$.

For $F$, remember that the energy that is transported upwards through the atmosphere is absorbed energy from the sun, times a factor of $\sim$2 to account for the internal heat that Jupiter self-generates because it is still (after all these eons) gravitationally contracting and converting gravitational potential energy into heat.

Is your computed dimensionless quantity tiny? You may find Figure 1.26 of Jupiter's thermal atmospheric structure in the Course Reader helpful for some actual numbers.

**Problem 3.** The Four Seasons

Consider a simplified model for the seasonal variation of the temperature of an atmosphere. Assume that the temperature of each vertical column is independent of height and neglect horizontal heat transport. In this scheme, the instantaneous temperature as a function of time, $T(t)$, is determined by the absorption of a variable solar flux, $F(t) = F_0 + F_1 \cos \omega t$, where $F_0$ and $F_1$ are constants. The re-radiated flux into space is $F_{IR}(t) = \sigma T^4$. Here $\omega = 2\pi/P$ is the frequency over which the solar flux varies at some latitude on the obliquely tilted planet, and $P$ is the orbital period of the planet about the sun.

(a) Describe why

$$\int_0^\infty dz\ \rho c_p \frac{\partial T}{\partial t} = F(t) - \sigma T^4 \tag{3}$$

where $c_p$ is the specific heat [erg/g/K] of atmospheric gas, $z$ is the height above the planet's surface, and $\rho$ is the mass density of the atmosphere. You should explain the physical significance of each term.

(b) Linearize this equation to obtain

$$I \frac{\partial T_1}{\partial t} + 4\sigma T_0^3 T_1 = F_1 \cos \omega t \tag{4}$$

where $I = \int_0^\infty dz \rho c_p$ is the thermal inertia, $T = T_0 + T_1$ and $\sigma T_0^4 = F_0$. Here $T_1$, the (time-varying) perturbation temperature, is small compared to (the time-independent) $T_0$. "Linearize" means "Taylor expand $T$, drop the zeroth order terms, and keep only the first order perturbation terms."

(c) Try a solution of the form $T_1/T_0 = \Delta \cos(\omega t - \phi)$. Prove that

$$\tan \phi = \frac{\omega I T_0}{4F_0}\,, \tag{5}$$

and that

$$\Delta = \frac{\cos \phi}{4} \frac{F_1}{F_0}\,. \tag{6}$$

(d) State the physical significance of

$$\frac{IT_0}{4F_0}\,. \tag{7}$$

Estimate the value of this quantity (be sure to give units) for a region on the Earth's surface at the latitude of, say, the Mojave desert. You should make the estimate by estimating each of the terms $I$, $T_0$, and $F_0$.

(e) Estimate $F_1/F_0$ for the Mojave desert.

(f) Evaluate $\phi$ and $\Delta$ using (d) and (e), and insert into your solution for the fractional seasonal temperature variation, $T_1/T_0$. Comment on the relation of your results to reality; give the physical significance of both $\phi$ and $\Delta$.

**Problem 4.** Eternal Sunshine of the Venusian Mind

Explain quantitatively why the Venusian surface does or does not experience the seasons over a Venusian year.

If you are going to use temperature as a variable, be clear about the distinction between actual temperature and effective temperature (in other words, be sure you understand the greenhouse effect).

Feel free to look up any numbers that you need.