Final Review Questions.

1. Compare the atmospheric scale height of the Sun, a brown dwarf, a $50M_\odot$ main sequence star, a post-MS giant star, a white dwarf, and a neutron star.
2. Given a density ρ and temperature T, what is the pressure of a fully ionized gas that is 10% H by mass, 50% He by mass, 20% C by mass, and 20% O by mass?
3. What does the statement that "self-gravitating systems have negative heat capacity" mean in words?
4. Why do hydrogen-burning main sequence stars have a minimum and maximum mass?
5. Why is thermal conduction so inefficient at transporting energy compared to radiative diffusion in main sequence stars, even though gas pressure generally dominates over radiation pressure?
6. Under what physical conditions does Thomson scattering occur? What about free-free absorption? What about H^- absorption?
7. Show that, for stars in which Thomson scattering dominates radiative diffusion, $L \propto M^3$.
8. Show that, for stars in which free-free absorption dominates radiative diffusion, $L \propto M^5$, assuming constant central temperatures.
9. In words, what physical conditions cause convection to set in? Explain then why pre-MS stars undergoing KH contraction are initially fully convective. Also explain why low mass MS stars are fully convective, why solar-type stars have convective envelopes, and why higher mass MS stars have convective cores.
10. Estimate the convective velocity v_c (in cm/s) in the solar convection zone.
11. What kind of stars (and what stellar masses) are well described using a $n = 3/2$ polytrope from center to surface? What about a $n = 3$ polytrope?
12. How does effective temperature for a fully convective star depend on luminosity?
13. Recall the early stages of star formation, in which a diffuse cloud of gas collapses isothermally and eventually fragments into little clumps. Assuming the Sun formed from one of these clumps with $T = 10$ K (the temp. of the original diffuse cloud), what was the density of this clump just as it started to form?
14. What is different between a star undergoing Kelvin-Helmholtz contraction and a star on the main sequence? What is the same?
15. Consider a proto-star of $M = M_\odot$ undergoing KH contraction, such that its radius goes from $R = 10R_\odot$ to $R = 2R_\odot$.
    (a) What is the change in temperature during this time?
    (b) Assuming $L = L_\odot$ once the radius equals $R = 2R_\odot$, what is its remaining KH timescale?
16. Go back to Worksheet #5 and do it again (without looking at solutions first)! Include the proper relations for the various parts of the HR diagram (i.e. low mass, fully-convective stars; low mass, radiative diffusion dominated stars; higher mass, radiative diffusion dominate stars; very high mass, radiation pressure dominated stars).

17. Why will fusing two uranium nuclei not produce any energy, but rather *require* extra energy? Yet natural uranium is available, and so must be produced in stars – how?

18. Explain, using words, why the typical energy of particles participating in hydrogen fusion reactions is neither the typical thermal energy nor the Gamow energy. What does the Gamow energy represent?

19. Why is it that at $T \sim 10^7$ K, the proton-proton chain and the CNO cycle produce roughly the same amount of energy, even though the CNO cycle requires much higher Gamov energies (and thus has much lower quantum tunneling probabilities)?

20. Why does the central temperature of the Sun increase slightly during its MS lifetime?

21. What are the various ways of quantifying whether a gas is degenerate or not?

22. What is the typical energy per particle of a non-relativistic degenerate gas with temperature $T = 0$ and density n.

23. Physically, why is there a maximum central temperature that is achievable as a star contracts and degeneracy pressure becomes important? Given that $T > 2 \times 10^6$ K is needed for H fusion, estimate the minimum mass of H-burning MS stars.

24. At what temperature is a gas of hydrogen 10% ionized, assuming a total particle density of $n = 10^{17}$ cm^{-3}?

25. Go back to Worksheet #9 and do it again! Make sure to explain why a solar MS star evolves the way it does off the MS. Also explain why an AGB star sheds so much mass.

26. Derive the mass-radius relationship for white dwarfs.

27. What is the physical meaning of the Chandrasehkar maximum mass for a degenerate object? Why does it happen?

28. Chart the various stages of stellar evolution for a high mass star.

29. At what temperature do thermal neutrinos start to get produced, significantly affecting the energy balance of a star? What reaction governs this?

30. Why do the late stages of stellar evolution for massive stars occur *very* quickly?

31. How is hydrogen fusion different from fusion of very heavy elements (i.e. like Si)?

32. Explain the physical processes involved in a core-collapse SN (i.e. how and why is the bulk of a star unbound and ejected outward).

33. Why does the Fe core go from KH contraction to free-fall collapse?

34. Why is the typical energy released in a core-collapse SN always $\sim 10^{53}$ ergs, regardless of the mass of the original star? What fraction of this energy goes into neutrinos, KE, and optical luminosity?

35. Where does a SN's optical luminosity originate from? Estimate the optical luminosity of a core-collpase SN.

36. How are elements heavier than Fe formed in SN explosions?

37. How are white dwarfs and neutron stars similar? How are they different?

38. What major theoretical hurdle prevents us from fully understanding the structure of NSs?

Sample Problems

39. **The Chandrasekhar Mass (15)**

    Consider a star supported solely by degeneracy pressure. Recall that such stars have the remarkable property that they have a maximum mass: hydrostatic equilibrium is not possible above a certain mass M_{max}. Derive an order of magnitude estimate of M_{max} for a white dwarf composed of carbon nuclei and electrons. You should give an expression for M_{max} in terms of fundamental constants; also evaluate your expression for M_{max} (in $M_\odot$).

40. **Cold Fusion! (14)**

    In thermonuclear fusion in stars, the gas must be sufficiently hot in order for fusion to occur. This is because high temperatures are needed to effectively tunnel through the Coulomb barrier. Recall from class that the approximate quantitative condition for fusion to occur is that the de Broglie wavelength λ must be comparable to the classical turning point r_c (the classical distance of closest approach) of two ions interacting via the Coulomb force.

    Now consider a low temperature degenerate gas of fermions. Each ion has Z protons and mass number A. The ion number density is n_i and the ions are non-relativistic. Because fermions have kinetic energy even at zero temperature it is still possible for the ions to tunnel through the Coulomb barrier in a cold degenerate gas. This leads to cold fusion, for which the technical name is pychnonuclear reactions (realistic calculations take into account that the ions are in lattice at low temperatures, but we won't worry about that).

    a) For ions of total energy E interacting solely via the electric force, calculate the closest the two ions can get to each other, the classical distance of closest approach r_c, because of the repulsive Coulomb interaction. (3)

    b) Using a), show that an approximate condition for significant tunneling through the Coulomb barrier – and thus for fusion – in a cold degenerate gas is that the ion number density exceed a critical value. You should derive an expression for the critical density n_i in terms of fundamental constants and the proton number Z and mass number A of the ions. (8)

    c) For Carbon (Z = 6; A = 12), what is the critical density n_i (in cm^{-3})? In what type of star might the conditions for pychnonuclear reactions be realized? (3)