Homework #2

1. I mentioned in class that there are two ways to estimate the energy carried by convection. The first is that the energy flux is $F_c \approx 1/2 \rho v_c^3 \equiv F_{c,1}$ where v_c is the characteristic velocity of the convective motions. This is the KE flux carried by moving blobs. The other estimate is that $F_c \approx \rho \Delta E v_c \equiv F_{c,2}$ where ΔE is the difference in the thermal energy of a rising hot blob (or sinking cool blob) relative to the background star (where E is per unit mass). I claimed in lecture that these two expressions are equivalent, to order of magnitude (which is the accuracy of mixing length theory). Show that this is correct.

2. The solar convection zone contains very little mass (only $\approx 2\%$ of the mass of the sun). Consider a model in which we neglect the mass of the convection zone in comparison to the rest of the sun. Model the convection zone as a polytrope with $P = K\rho^\gamma$. The radius of the base of the convection zone is R_c.

   a) Solve for the density, temperature, and pressure as a function of radius in the convection zone. Do *not* assume that the convection zone is thin (i.e., even though M_r = constant = M, because r changes significantly in the convection zone, do not assume that the gravitational acceleration is constant).

   b) In detailed solar models, the pressure at the base of the convection zone is $\approx 5.2 \times 10^{13}$ dyne/cm^2 and the density is $\rho \approx 0.175$ g cm^{-3}. Estimate the radius of the base of the convection zone R_c. Compare this to the correct answer of $R_c \approx 0.71 R_\odot$.

   c) In your model, what is the temperature of the sun at $0.99 R_\odot$, $0.9 R_\odot$, and at the base of the solar convection zone. This gives you a good sense of how quickly the temperature rises from its surface value of ≈ 5800 K as one enters the interior of the sun.

   d) Using a simple publically available stellar structure code, I find that the zero-age main sequence (ZAMS) solar model has the following properties (ZAMS is when the star first starts to undergo fusion, so no H has yet been converted into He; this is $t = 0$ in stellar models, vs. $t \approx 4.5$ billion years for the sun today).

$\log(T_{eff})$	$\log(L/L_\odot)$	$\log(T_c)$	$\log(\rho_c)$	$\log(R/R_\odot)$
3.7256	-0.3287	7.1287	1.8444	-0.0886

   This calculation doesn't quite get the luminosity, radius, etc. of the sun correct because it uses simple analytic opacities and energy generation terms, but it's not bad.

   For the same code, if I turn off convection, so that energy is always transported by radiation even when the solution is in reality convectively unstable, i find the following solar model

$\log(T_{eff})$	$\log(L/L_\odot)$	$\log(T_c)$	$\log(\rho_c)$	$\log(R/R_\odot)$
3.6978	-0.3292	7.1286	1.8443	-0.033

   The luminosity, central temperature, and central density do not change significantly (not surprising since the convection zone has so little mass). However, the radius of the model sun increases by about 14% and the effective temperature decreases as a result by about 7%.

   To understand this, consider the structure of a radiative atmosphere in which gas pressure dominates and the enclosed mass, enclosed luminosity and opacity are all constant (constant M_r and L_r are quite reasonable at large radii, while constant κ is only approximately valid and is made for simplicity). Use hydrostatic equilibrium and the radiative diffusion equation to derive a relationship betweeen pressure and density in such an atmosphere. Use this and your model atmosphere from a) to explain the numerical results above, namely that the radius of the sun would be larger if the outer convection zone were absent. Note: In class, I claimed that an $n = 3$ polytrope is a pretty good model of the radiative parts of the sun. This problem should help you see why this is a reasonable first approximation.

3. In class, we discussed Kelvin-Helmholz (KH) contraction of fully convective stars to the main sequence. As part of this calculation, we showed that moderately massive stars become radiative well before they reach the main sequence.

   a) Does the transition from convective to radiative first happen at the center of the star or the outside (i.e., is the transition inside-out or outside-in)? To determine this, consider where the star is likely to first become stable by the criterion $d\ln T/d\ln P|_{rad} < 2/5$. During KH contraction the energy source for the star (gravity) depends only weakly on density and temperature so assume that $L_r/M_r \sim$ constant, independent of position in the star.

   In the rest of this problem, we will derive the KH contraction of a radiative star. Assume for simplicity that electron scattering dominates the opacity of the star.

   b) Derive R(t,M) for radiative stars undergoing KH contraction.

   c) If stars reach the main sequence when $R/R_\odot \approx M/M_\odot$, what is the time to reach the main sequence, t_{MS}, as a function of stellar mass?

   d) Compare your result from c) to the result derived in lecture for when stars become radiative after evolving down the Hayashi track. Do stars spend most of their pre-main sequence evolution as radiative or convective objects? Does this depend on the mass of the star?