EQUATIONS from Astro 7A

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2 Celestial Mechanics

Law of Gravity:

$$F = GM_1M_2/r^2$$

Equation of Ellipse:

$$r = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{a(1-e)(1+e)}{1+e\cos\theta}$$

Perihelion: θ=0
Kepler's Third Law:

$$P^2 = \frac{4\pi^2}{G(M_1+M_2)}a^3$$

Potential energy: $U = -GMm/r$
Energy:

$$E = 1/2\mu v^2 - G\frac{M\mu}{r}$$

Angular Momentum:

$$\vec{L} = \mu\vec{r}\ X\ \vec{v}$$

where $\mu \equiv \frac{m_1m_2}{m_1+m_2}$

Virial Theorem:

$$< U >= -2 < K >$$

3 Magnitudes and Light

$$m_1 - m_2 = -2.5\log_{10}(\frac{F_1}{F_2})$$

Distance modulus:

$$m - M = 5 \log_{10} d - 5$$

distance in pc:

$$d = 1/\pi''$$

Wien's law:

$$\lambda_{max} T = 2.898 \times 10^{-3} \mathrm{mK}$$

Blackbody Radiation Flux

$$F_{surf} = \sigma T^4$$

Blackbody Energy Density

$$U = aT^4$$

Planck Fn.

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

$$E = h\nu$$

$$p = E/c$$

Diffraction

$$\theta = \lambda/D$$

4 Relativity

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$$

$$t_1^{'} - t_2^{'} = \frac{(x_2 - x_1)u/c^2}{\sqrt{1 - u^2/c^2}}$$

$$\Delta t_{\text{moving}} = \frac{\Delta t_{\text{rest}}}{\sqrt{1 - u^2/c^2}}$$

$$L_{\text{moving}} = L_{\text{rest}}\sqrt{1 - u^2/c^2}$$

$$E_{\text{rest}} = mc^2$$

$$E^2 = p^2c^2 + m^2c^4$$

Doppler Effect (v$\ll$c)

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = v_r/c$$

Doppler Effect (v$\sim$c)

$$\nu_{\text{obs}} = \frac{\nu_{\text{rest}}\sqrt{1 - u^2/c^2}}{1 + v_r/c}$$

Redshift, (z)

$$z = \Delta\lambda/\lambda_{rest}$$

5 Atomic Structure

Hydrogen energy levels:

$$E_n = -\frac{\mu e^4}{2\hbar^2 n^2} \ \ (\text{cgs}) = 13.6 \text{ eV}/n^2$$

$$\ell = 0, 1, 2 \cdots n-1$$

$$m_\ell = -\ell, -\ell+1, \cdots, 0, \cdots, \ell-1, \ell$$

$$L = \sqrt{l(l+1)}\hbar$$

$$E = h\nu$$

6 Telescopes

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta = 1.22\lambda/D$$

Solid Angle

$$\Omega \equiv 4\pi * Area/4\pi r^2$$

7 Binary Stars

$$M_1 a_1 = M_2 a_2$$

$$M_1 v_1 = M_2 v_2$$

$$P^2 = 4\pi^2 a^3/[G(M_1 + M_2)]$$

$$a = a_1 + a_2 = \frac{P}{2\pi}(v_1 + v_2)$$

$$\frac{m_2^3}{(m_1 + m_2)^2} \sin^3 i = \frac{P}{2\pi G} v_{1r}^3$$

8 Hot Gas and Stellar Spectra

$$n_v \mathrm{d}v = n(m/2\pi kT)^{3/2} e^{-1/2mv^2/kT} 4\pi v^2 \mathrm{d}v$$

$$n_{v_x} \mathrm{d}v_x = n(m/2\pi kT)^{1/2} e^{-mv_x^2/2kT} dv_{\mathrm{x}}$$

$$N_b/N_a = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}}$$

$$Z = \sum g_i e^{-E_i/kT}$$

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} (\frac{(2\pi m_e kT)^{3/2}}{h^3}) e^{-\chi_i/kT}$$

9 Stellar Atmospheres

$$I = I_o e^{-\tau}$$

$$\tau = \int \kappa \rho dx = \int n\sigma dx$$

$$\mathrm{d}I = j_\lambda \rho \mathrm{d}s - \kappa_\lambda \rho I \mathrm{d}s$$

10 Stellar Structure

Hydrostatic Equilibrium:

$$dP/dr = -GM_r\rho/r^2$$

Mass Conservation:

$$dM_r/dr = 4\pi r^2\rho$$

Energy Generation:

$$dL_r/dr = 4\pi r^2\rho\epsilon$$

Radiation Transport of energy:

$$dT/dr = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2}$$

Adiabat:

$$PV^\gamma = const$$

Collision (or reaction) rate between particles x and i:

$$r = n_x n_i \sigma v$$

pp-chain first step:

$$H + H - - > D + e^+ + \nu_e$$

11 Constants

c = 3.00×10^8 m/s
h = 6.623×10^{-34} J s
$\sigma = 5.67 \times 10^{-8}\ W/m^2K^{-4}$
$a = 7.56 \times 10^{-16} Jm^{-3}K^{-4}$
$L_\odot = 3.90 \times 10^{26}$ W
$M_\odot = 1.99 \times 10^{30}$ kg
$G = 6.67 \times 10^{-11} Nm^2/kg^2$
$k = 1.38 \times 10^{-23} J/K$
$1eV = 1.6 \times 10^{-19} J$