

NAME -----

## **Final Exam**

### **Astronomy 7B: Introduction to Astrophysics**

Explain all your answers. You may write on these pages, or use your own. You have 3 hours.

# 1 General Relativity

A massive, spherical star has mass,  $M$ , radius,  $R$ , and a surface effective temperature,  $T$ . It radiates like a blackbody.

- a) What is the luminosity of the star, in its frame?
- b) Viewed from infinity, the wavelengths are different from those emitted due to gravitational redshift. Using Wien's law, what is the temperature measured by an observer at infinity,  $T_\infty$ ? Is it lower or higher than  $T$ ?
- c) A telescope with diameter,  $D$ , located a (long) distance,  $d$  away will gather how much energy per second,  $dE/dt$ ?

## 2 Globular Clusters and Tidal Forces

A spherical globular cluster has a mass,  $M_{gc}$ , and a radius,  $R$ . It has constant density throughout its interior and is held together by its own self-gravity. A supermassive black hole with mass  $M_{BH}$  is located outside the cluster, a distance,  $r$ , from its center.

a) First consider the self-gravity of the globular cluster, and consider it composed of two halves (one half facing the BH and the other facing away). Write an expression for the force of gravity,  $F_{\text{binding}}$ , exerted by one half on the other half of the globular cluster. Treat the two halves as point masses. This force holds the globular cluster together.

b) Write down an expression for the tidal force, i.e., the difference between the forces exerted by the BH on each half of the globular cluster,  $\Delta F$ .

c) Estimate the critical distance,  $r_{\text{crit}}$ , between the black hole and globular cluster at which the globular cluster breaks up.

### 3 Contracting Cloud

A cloud of pure hydrogen atoms (each of mass  $m_H$ ) has uniform density but is slowly contracting, in equilibrium at all times. It has a total mass,  $M$ . At time  $t$ , it has radius,  $R(t)$  and the rate of contraction is  $\frac{dR}{dt}$ . (Assume the cloud is not rotating.)

a) Derive an expression for the potential energy of this cloud,  $U$ , and calculate the rate of change of the potential energy with time,  $\frac{dU}{dt}$ , at time  $t$ .

b) From the virial theorem, what is the thermal energy,  $E_{th}$ , of the gas inside the cloud, for a given  $M$  and  $R$ ?

c) Use your answers from parts (a and b) to get an expression for the luminosity of this cloud due to contraction in terms of the physical parameters,  $M$  and  $R$ , and the specified rate of contraction,  $\frac{dR}{dt}$ .

d) What is the effective temperature of the surface,  $T_{\text{eff}}$ , at time,  $t$ ?

## 4 Spherical Galaxy

An elliptical galaxy is spherical and contains stars and dark matter, both extending to a maximum distance,  $R_{\text{gal}}$ . The stars have uniform density,  $\rho_{\text{stars}}$  but the dark matter has density that depends on distance,  $r$ , from the center,

$$\rho_{\text{dark}}(r) = k/r^2$$

, where  $k$  is a constant.

- a) What is the mass interior to a radius,  $M_r$ ?
- b) What is the approximate velocity,  $v$ , of a star at a distance,  $r$ , from the center?
- c) Sketch  $v(r)$ .
- d) An absorption line from the galaxy has rest wavelength,  $\lambda_o$ . What is the approximate width of that line?
- e) The center of that absorption line has an observed wavelength that is 10% greater than the rest wavelength. How far away is the galaxy?

## 5 Friedmann Equation

Consider a matter-dominated universe that has no Cosmological Constant ( $\Lambda = 0$ ) and which is flat. The universe has a mass density (luminous and dark matter combined) of  $\rho_0$  today.

a) Start with the Friedmann Eqn. and derive an expression for the Hubble constant today,  $H_0$  in terms of  $\rho_0$ .

b) Derive an expression for the scale factor,  $R(t)$  in terms of  $\rho_0$ .

c) A certain galaxy has redshift,  $z=4$ . What was the age of the universe when it emitted its light?

d) What was the mass density,  $\rho(t)$ , when that galaxy emitted its light? (constants.)

e) The galaxy emits light at all wavelengths from its stars, and it emits Lyman alpha in emission from hot gas. Along the same line of sight are four cold clouds of neutral hydrogen at redshifts  $z = 3.8, 3.9, 4.1$  and  $4.2$ . Which cloud is farthest from us? Sketch the spectrum that we receive from that galaxy, in the vicinity of the Lyman alpha line.

Friedman Eqn:

$$\left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho R^2 = -kc^2$$

## 6 Tanning

Summer is here. You go to the beach at 4pm to daydream about the fate of our expanding,  $\Lambda$ -dominated universe, and also to get a tan. Of course, the Earth's atmosphere scatters incoming sunlight due to Rayleigh scattering by air molecules. When the Sun is overhead, 90% of the light having wavelength 500 nm (green) reaches the ground.

At 4pm the Sun is about 60 degrees from overhead. **At ultraviolet wavelengths of 250 nm (that gives you a tan), what fraction,  $f$ , of the light reaches the ground?** You may assume that the Earth's atmosphere is a flat slab, without curvature. (You may leave your answer with numbers, if you want to avoid using a calculator.)

## 7 Luminosities and Velocity Dispersions of Galaxies

For elliptical galaxies, their luminosities,  $L$ , correlate strongly with the velocity dispersion,  $\sigma$ , of the stars (as measured in the widths of absorption lines). Derive the relationship between  $L$  and  $\sigma$ . You may assume that ellipticals have similar surface brightnesses and that the amount of dark matter is proportional to the mass of visible stars.

Hint: You can use proportionalities, and neglect constants in order to go faster. But explain each step with a few words.