Lecture 27 Stability of Molecular Clouds

1. Collapse and Fragmentation
2. The Virial Theorem and Stability
3. The Role of Magnetic Fields

References

- Shu II, Ch. 24 (magnetic virial theorem)
- Palla and Stahler, Ch. 9
- McKee, Crete 1999:

http://www.cfa.harvard.edu/events/1999crete

1. Collapse and Fragmentation

First ask*, how do cores condense out of the lower density regions of GMCs?*

- The conventional wisdom is local ***gravitational instabilities*** in a globally stable GMC via the classical ***Jeans instability***.
- Consider a uniform isothermal gas in hydrostatic equilibrium with gravity balanced by the pressure gradient. Now suppose a small spherical region of size r is perturbed

$$\rho_0 \rightarrow X\rho_0 \quad \text{with} \quad X > 1$$

The over-density $X\rho_0$ generates an outward pressure (force per unit mass):

Simple Stability Criterion

$$F_p \sim |\nabla p| / \rho_0 \sim Xc^2 / r$$

The increased density leads to an inward gravtitational force per unit mass

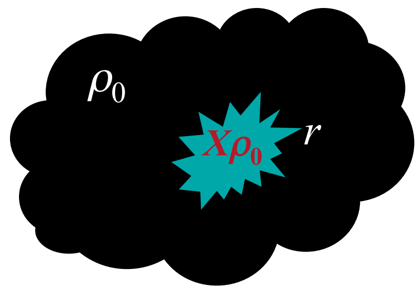
$$F_G \sim GM(r)/r^2 \sim GX\rho_0 r.$$

Gravity wins out if

$$r^2 > \frac{c^2}{G\rho_0} \text{ or } r > \frac{c}{\sqrt{G\rho_0}}$$

The right hand side is essentially the **Jeans length**:

$$\lambda_J = c\sqrt{\frac{\pi}{G\rho_0}}$$



Turbulent Jeans Length

An assumption made implicitly in the above “derivation” is that molecular clouds are quiescent, which ignores turbulence.

Turbulence provides an additional pressure support. The associated supersonic motions must be dissipated by shocks before the collapse. Turbulent pressure is usually expressed by replacing the sound velocity *c* by

$$c^2 = \frac{kT}{m} + \sigma_{\text{turb}}{}^2$$

As the gas condenses, the Jeans length (inversely proportional to $\rho^{-1/2}$) decreases, and this suggests ***fragmentation*** (Hoyle 1953 ApJ 118 513), i.e., the collapse occurs on smaller and smaller scales.

Does Fragmentation Occur?

1. The *dispersion relation* for a perturbation $\delta\rho \sim e^{i(\omega t - kx)}$ in the Jeans problem is (c.f. Problem 3)

$$\omega^2 = c^2 k^2 - 4\pi G\rho_0$$

or

$$\omega^2 = c^2 (k^2 - k_J^2), \quad k_J^2 = 4\pi\ G\rho_0 / c^2$$

2. $k^2 - k_J^2 < 0$ makes ω imaginary:
   - Exponential growth occurs for $k < k_J$
   - Growth rate $-i\omega$ increases monotonically with *decreasing k,* which implies that
     - the longest wavelength perturbations (largest mass) grow the fastest
     - fast collapse on the largest scales suggests that ***fragmentation is unlikely***, disagreeing with Hoyle (Larson 1985 MNRAS 214 379)

Jeans Length in Two Dimensions

Consider an alternate model, a thin sheet of surface density Σ.The dispersion relation is

$$\omega^2 = c^2 k^2 - 2\pi\ G\ \Sigma\ |k|$$

or

$$\omega^2 = c^2 (\ k^2 - k_c\ |k|\ ) \qquad \text{with} \qquad k_c = 2\pi\ G\ \Sigma / c^2.$$

Exponential growth occurs for $k < k_c$ with growth rate

$$-i\ \omega = c\ (\Sigma\ k_c k - \ k^2\ )^{1/2}$$

which is a maximum at $k_f = k_c/2 = \pi\ G\ \Sigma / c^2$

Compared with the 3-d analysis, the maximum growth rate occurs on an intermediate (smaller scale) that may favor fragmentation. For a non-uniform cloud, other lengths can enter, e.g., the scale height of the sheet.

Be careful using the simple Jeans result!

2. The Virial Theorem and Stability

The derivation of the virial theorem starts with the equation of motion for the *macroscopic* velocity of The fluid after averaging over the random thermal velocities. Ignoring mechanical dissipation, the MHD equations of motion with gravity are (Shu II.24):

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla}P - \rho\vec{\nabla}\phi + \vec{\nabla}\cdot\overleftrightarrow{T}$$

$$T_{ij} = B_i B_j / 4\pi - B^2 \delta_{ij} / 8\pi$$

$$\vec{\nabla}\cdot\overleftrightarrow{T} = -\frac{1}{8\pi}\nabla B^2 + \frac{1}{4\pi}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$$

Our perspective is that these equations contain too much information for quick comprehension, so one takes moments, specifically the first moment:

Virial Theorem: LHS Analysis

Take the dot product with ***r*** and integrate the equation of motion over a finite volume, and analyze the right and left hand sides separately.

The LHS side yields two terms since:

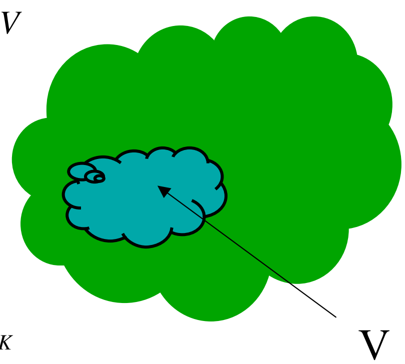
$$\int_V \rho \ddot{\vec{r}}\cdot\vec{r}\, dV = \frac{1}{2}\frac{D^2}{Dt^2}\int_V (\vec{r}\cdot\vec{r})\,\rho\, dV - \int_V (\vec{v}\cdot\vec{v})\,\rho dV$$

and

$$\frac{D^2}{Dt^2}\left(\vec{r}\cdot\vec{r}\right) = 2\left(\dot{\vec{r}}\cdot\dot{\vec{r}} + \vec{r}\cdot\ddot{\vec{r}}\right)$$

and finally

$$\frac{1}{2}\frac{D^2}{Dt^2}\left(\int_V r^2\rho\, dV\right) - \int_V v^2\rho\, dV = \frac{1}{2}\frac{D^2 I}{Dt^2} - 2E_K$$



The 1st term vanishes for a static cloud.

Virial Theorem: RHS Analysis

- The RHS yields volume and surface terms

$$\int_V 3P\,dV + \int_V \frac{B^2}{8\pi} dV - \int_V \left(\vec{r}\cdot\vec{\nabla}\phi\right) dm$$

$$-\int_S \left(P + \frac{B^2}{8\pi}\right)\vec{r}\cdot d\vec{S} + \frac{1}{4\pi}\int_S \left(\vec{r}\cdot\vec{B}\right)\left(\vec{B}\cdot d\vec{S}\right)$$

- The 3rd term becomes the gravitational energy.
- The 1st and 4th terms measure the difference between the external and mean internal pressures.
- The other terms are transformed into magnetic pressure and tension.

The Virial Theorem: Final Result

$$\frac{D^2 I}{Dt^2} = 2E_{\mathrm{K}} + 3\left(\langle P\rangle - P_{\mathrm{ext}}\right) V + W + M_B$$

where

$$M_B = \int dV \frac{B^2}{8\pi} + \frac{1}{4\pi}\int d\vec{S}\cdot\vec{B}(\vec{B}\cdot\vec{r}) - \frac{1}{8\pi}\int d\vec{S}\cdot\vec{r}\ B^2$$

For a static cloud, we can solve for the external pressure

$$P_{\mathrm{ext}}\,V - \langle P\rangle\,V = \frac{2}{3}E_{\mathrm{K}} + \frac{1}{3}W + \frac{1}{3}M_B$$

and then insert approximate expressions in the RHS.

P_{ext} becomes a function of the global variables of the volume V and of the gas inside. If there is no rotation, $E_{\mathrm{K}} = 0$.

Simplest Application of the Virial Theorem

Consider a spherical, isothermal, unmagnetized and non-turbulent cloud of mass *M* and radius *R* in equilibrium :

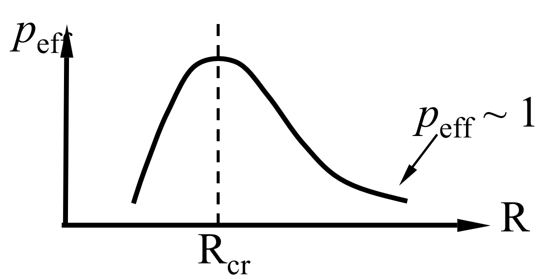
$$4\pi R^3 P_{\rm ext} = 3Mc^2 - \frac{3}{5}\frac{GM^2}{R}$$

where we have used $P = \rho c^2$. The two terms on the right give the integrated effects of internal pressure and of self-gravity. When we ignore gravity, we can verify that the external and internal pressures are the same. With gravity, the second term lowers the external pressure needed to confine the gas inside the volume *V.*

Next, divide by $4\pi R^3$ and plot the RHS vs. *R*

Simplest Application of the Virial Theorem

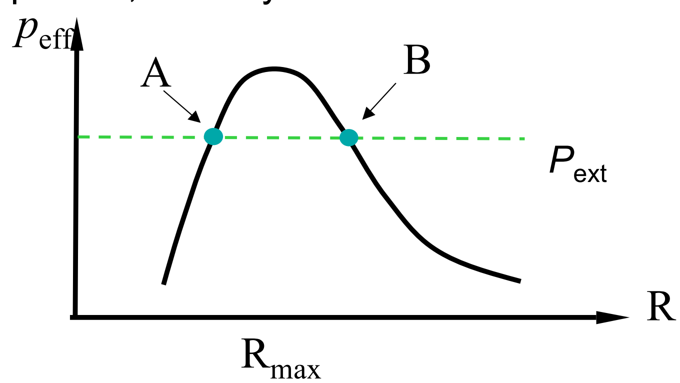
$$P_{\rm ext} = \frac{3c^2 M}{4\pi}\frac{1}{R^3} - \frac{3GM^2}{20\pi}\frac{1}{R^4} = p_{\rm eff}(R;M)$$



- There is a minimum radius below which this model cloud cannot support itself against gravity.
- States along the left segment, where $p_{\rm eff}$ *de*creases with decreasing *R* (compression), are unstable.
- There is no stable equilibrium for too high pressure.

Stable and Unstable Virial Equlibria

For a given but not too high external pressure, there are two equilibria, but only one is stable.



Squeeze the cloud at *B (*decrease *R)*
– Requires more pressure to confine it: re-expands (stable)

Squeeze the cloud at *A* (decrease *R)*
– Requires less pressure to confine it: contracts (unstable)

The Critical Pressure

Differentiating the effective pressure for fixed *M* and *c*

$$p_{\rm eff} = \frac{3c^2 M}{4\pi}\frac{1}{R^3} - \frac{3GM^2}{20\pi}\frac{1}{R^4}$$

gives the critical radius,

$$R_{\rm cr} \approx \frac{GM}{c^2}$$

The corresponding density and mass are of order

$$\rho_{\rm cr} \approx \frac{c^6}{M^2} \qquad M_{\rm cr} \approx \frac{c^3}{\sqrt{\rho_{\rm cr}}}$$

This results is essentially the same as the Jeans analysis, but the virial method provides a more secure derivation.

Fragmentation vs. Stability
Reconciliation of Jeans and Virial Analysis (JRG)

- Both methods yield a similar characteristic “Jeans” mass
  - The difference is in the initial conditions
    - Cloud scenario assumes a self-gravitating cloud core in hydrostatic equilibrium
      - The cloud may or may not be close to the critical condition for collapse
    - In the gravitational fragmentation picture there is no “cloud”
      - It cannot be distinguished from the background ***until it has started to collapse***

  Both perspectives may be applicable in different parts of GMCs
    - There are pressure confined clumps in GMCs
      - Not strongly self-gravitating
    - Cores make stars
      - Must be self-gravitating
      - If fragmentation produces cores these must be collapsing and it is too late to apply virial equilibrium

Fragmentation vs. Stability (JRG)

A relevant question is how quickly cores form (relative to the dynamical time scale)

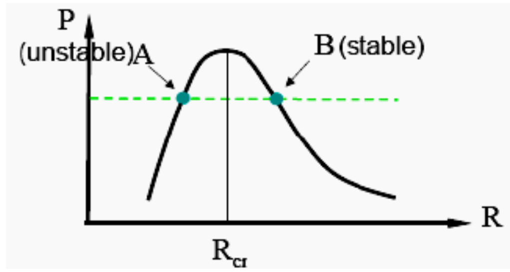
- NH_3 cores seem to be close to virial equilibrium but they could be contracting (or expanding) slowly

Statistics of cores with and without stars should reveal their ages

- If cores are stable, there should be many more cores without stars than with young stars
- Comparison of NH_3 cores and *IRAS* sources suggests ~ 1/4 of cores have young stellar objects (Wood et al. ApJS 95 457 1994), but Jijina et al. give a larger fraction (The corresponding life time is < 1 Myr (T Onishi et al. 1996 ApJ 465 815).

Contrast GMCs, which almost always have young stars and cores. Most cores have no stars and in some small clouds none.

Exact Hydrostatic Equilibrium



Assuming an isothermal sphere, the equilibrium state B can be obtained by solving a non-linear ordinary differential equation.

For a given mass and temperature, there is a maximum pressure for stability. Alternatively, given a value of the external pressure, there is a maximum stable mass. The solution of the Lane-Emden equation by Bonnor (MNRAS 116 351 1956) & Ebert (Zs. f. Ap 37 222 1955) gives the critical mass as

$$M_{cr} \cong 1.2 \frac{c^4}{G^{3/2} P_{ext}} = O(M_J)$$

3. Magnetic Fields and Stability

- Observations show that GMCs are large (10-100 pc), cool (~ 20 K), fairly dense (~ 10^2 - 10^3 cm^{-3}), and turbulent (Δv ~ 1 km s^{-1}) but ***gravitationally bound.***

- More detailed observations show that they contain small (~ 0.1 pc), cold (~ 10 K), dense (~ 10^4 cm^{-3}) and often turbulent regions called ***cloud cores***; they are also bound, but perhaps not always gravitationally.

- The cores may also contain newly formed stars and thus manifest both stability *and* instability, although on different spatial scales. To explore this situation, we re-examine the stability of molecular clouds taking into account ***magnetic fields***.

Virial Analysis of a Magnetized Cloud

Apply the virial theorem to an assumed equilibrium cloud

$$P_{\text{ext}} V - \langle P \rangle V = \frac{2}{3} E_{\text{K}} + \frac{1}{3} W + \frac{1}{3} M$$

Ignore rotation and turbulence ($E_K = 0$) and focus on the last (magnetic) term. In the spirit of the previous discussion we make several rough approximations:

1. The magnetization is uniform,
2. The field outside is dipolar
3. Magnetic flux is conserved, $\Phi \approx \pi R^2 B$, i.e., the cloud is a sufficiently good conductor to allow flux-freezing

$$\begin{aligned} 4\pi R^3 P &= 3c^2 M - \frac{3}{5}\frac{GM^2}{R} + \frac{1}{3} R^3 B^2 \\ &= 3c^2 M - \frac{3}{5}\frac{GM^2}{R} + \frac{1}{3\pi^2}\frac{\Phi^2}{R} \end{aligned}$$

numerical error
1/3 → 1/6

Virial Analysis for a Magnetized Cloud

$$P = \frac{3}{4\pi}\frac{Mc^2}{R^3} - \frac{3}{20\pi}\frac{GM^2}{R^4} + \frac{1}{12\pi^3}\frac{\Phi^2}{R^4}$$

numerical error
1/12 → 1/24

The magnetic and gravitational energies vary with R in the same way.
Since they stand in a constant ratio, once a cloud starts collapsing the frozen-in magnetic field cannot stop it.
Magnetically dominated clouds are stable, i.e, clouds whose mass is less than the value:

$$M_\Phi \equiv \frac{1}{\pi}\left(\frac{5}{9G}\right)^{1/2} \Phi$$

numerical error
5/9 → 5/18

More careful analysis, originally due to Mouschovias & Spitzer (ApJ 210 326 1974) leads to the slightly more accurate formula,

$$\boxed{M_\Phi \approx 0.13\, G^{-1/2}\Phi = 1.0 M_{\text{sun}} \left(\frac{B}{20\mu\text{G}}\right)\left(\frac{R}{0.1\,\text{pc}}\right)^2}$$

NB In some papers, 0.13 is replaced by $1/2\pi$ in the last formula.

Magnetically Critical Mass

This analysis leads to the following terminology:
- *Magnetically Sub-Critical Mass*: $M < M_\Phi$ - Magnetism stronger than gravity. The external pressure maintains the cloud in equilibrium but does not collapse it.
- *Magnetically Super-Critical Mass*: $M > M_\Phi$ - Magnetism weaker than gravity. The external pressure aids collapse.

The virial equation can be rewritten in terms of M_Φ:

$$P = \frac{3}{4\pi}\frac{Mc^2}{R^3} - \frac{3}{20\pi}\frac{G}{R^4}(M^2 - M_\Phi{}^2)$$

Measurements of magnetic fields in cores (Troland and Crutcher (2008; slide 24 below) Indicate that they are often on the margin between sub and super critical.

The Fate of Sub-Critical Cores

- Sub-critical cores may collapse on long timescales via ***ambipolar diffusion*** (see Shu II, Ch. 27).
- Gravity acts on all components of the interstellar cloud, ions as well as neutrals (the dominant component).
- The ions are tied to the magnetic field; they try to force the neutrals to follow via ion-neutral collisions.
- Over time, however, gravity drags the neutrals through the ions to feed the collapse at the center.
- The “drift” speed varies inversely with the electron fraction, so ambipolar diffusion is faster near the center compared to the core surface
- Alternatively, gas dynamic forces in a ***turbulent cloud*** may drive a cloud core into collapse

Ambipolar Diffusion Timescale

Assuming ideal MHD, the force per unit volume is balanced by the drag force,

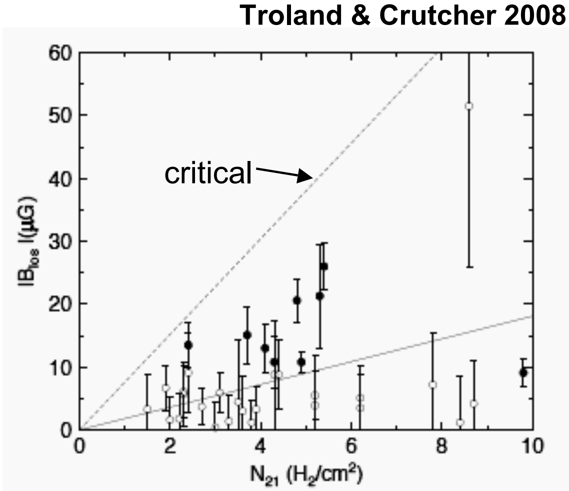
$$\frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} = \frac{m_i m_n}{m_i + m_n} K\, n_i n_n\, \vec{w}$$

where *i* and *n* stand for ions and neutrals, *K* is the rate coefficient for ion-neutral momentum-changing collisions, and **w** is the drift velocity of interest. One can easily check the dimensions of the right side, and that *w* is sensitive to: B, n_H^2 and x_e.

A crude estimate, based on replacing the LHS by $B^2/4\pi L$, and using B ~ 20 G, K ~ 10^{-9} cm^3 s^{-1}, H_2 as the neutral, and other typical core properties yields w << 1 km s^{-1}.

Core Magnetic Field Measurements

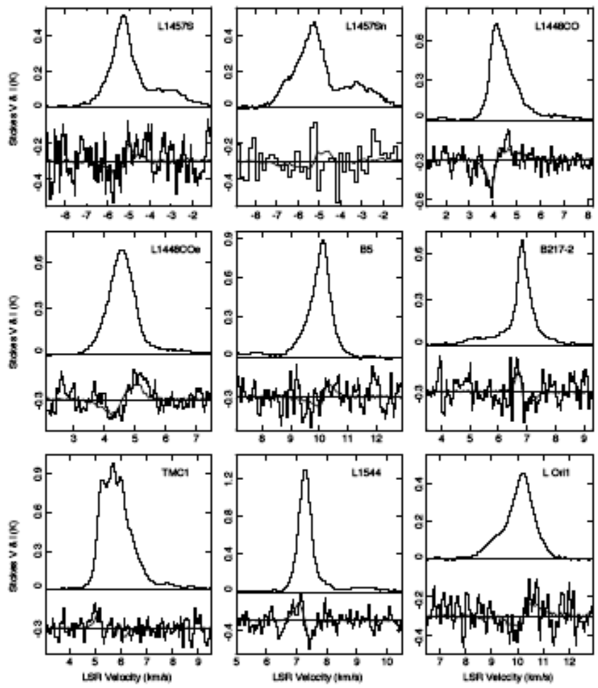
Troland & Crutcher 2008 astro-ph 0802.2253



- OH Zeeman measurements are most definitive but very difficult. Until recently, few results were reliable.
- Troland and Crutcher report 9 new "probable" 2.5σ detections after observing 34 dark clouds for 500 hours with Arecibo
- Error bars in the figure are 1σ.
- Corrected fit: M / M_Φ ~ 2

Fig. 2.— Results for B_{los} from the Arecibo dark cloud survey plotted against the H_2 column density ($N_{21} = 10^{-21}N$). The 9 probable detections (see text) are plotted as filled circles, while non-detections are plotted as open circles. Error bars are 1σ. The solid line is the weighted mean value for the mass to flux ratio with respect to critical inferred from the Zeeman B_{los} data with no geometrical correction; $\lambda \approx 4.8\pm0.4$. After geometrical corrections (see text), $\lambda_c \approx 2$, or slightly supercritical. The dashed line is the critical mass to flux ratio.

OH Spectra for Troland-Crutcher Detections

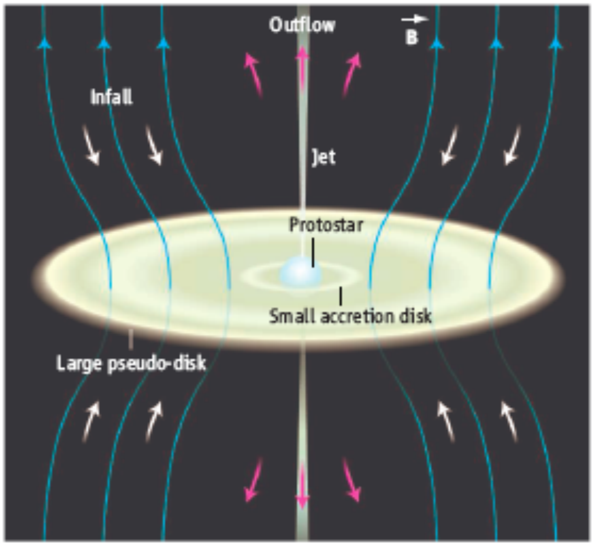


Discussion of Troland & Crutcher 2008

- Arecibo (3') observations of OH 1665, 1667 line shapes added and subtracted to give Stokes $I(\nu)$ and $dI(\nu)/d\nu$
- B_{los} for 34 dark cloud cores; 9 "probable" new detections
- Require geometrical correction of ~ 3 to get total *B.*
- OH/H_2 assumed to be 8×10^{-8} to obtain H_2 column.
- Core mass = $2.8 m_H \, \pi R^2 \, N(H_2)$
- Cores are mildly supersonic: Δv_{FWHM} = 0.7 km/s; other average properties are:

  $B_{los} \sim 8\ \mu G$, $R \sim 0.3$ pc, $N(H_2) \sim 4 \times 10^{21} cm^{-2}$
- Main result: $M/M_{\Phi} \sim 1.5$ (with geometrical correction of 3)
- $E_{turb} > E_{grav} > E_{mag}$ with each ratio ~ 2.
- Technical issues: large uncertainties (error bars), cores barely resolved in OH, which is a low-density tracer.
- Heterogeneous sample: cores come from a variety of molecular clouds

Magnetically Dominated Core Collapse

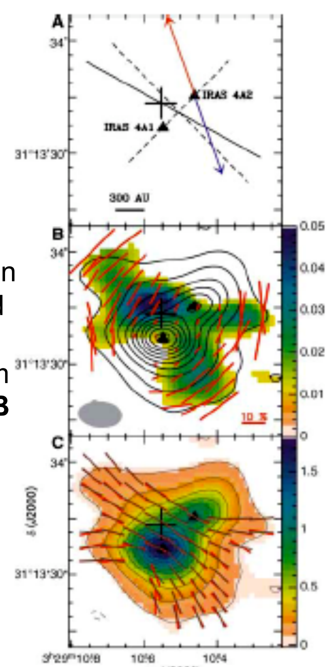


Crutcher, Science 313 771 2006

Magnetically Dominated Core Collapse

SMA ~ 1" observations of Girart et al. Science 313 812 2006

Physical Basis
• elongated grains spin about short axis and align with **B**
• radiate in continuum with polarization ⊥ **B**



NGC 1333 IRAS 4A
blue-red: outflow directions
solid line; B-field direction
dashed lines: envelope axis

contours: 877μm dust emission
colors: polarized emission
red bars: polarization degree

colors: 877 μm dust emission
red bars: measured B-field
grey bars: best fit parabolic model

Note resemblance to the expected hourglass shape

Summary: Magnetic Fields in Dense Cores

- The observations show that magnetic fields play a significant role in molecular clouds.
- Incomplete OH Zeeman measurements as of 2008 gave mass-to-flux ratios ~ critical and possibly larger.
- Turbulence must also play a role since the turbulent energy is somewhat larger than gravitational.
- Polarization maps show (ordered?) magnetic fields threading molecular clouds over large scales.
- Magnetic fields can shape a cloud core so that it tends to collapse in the direction of the field.
- The high angular resolution sub-mm polarization map of NGC 1333 IRA4 shows the expected hourglass shape

Flash! Crutcher et al. (ApJ 692 844 2009) reports measurements for four core-halo ratios of the mass-to-flux ratio for dark clouds that support the dominant role of turbulence in core formation.

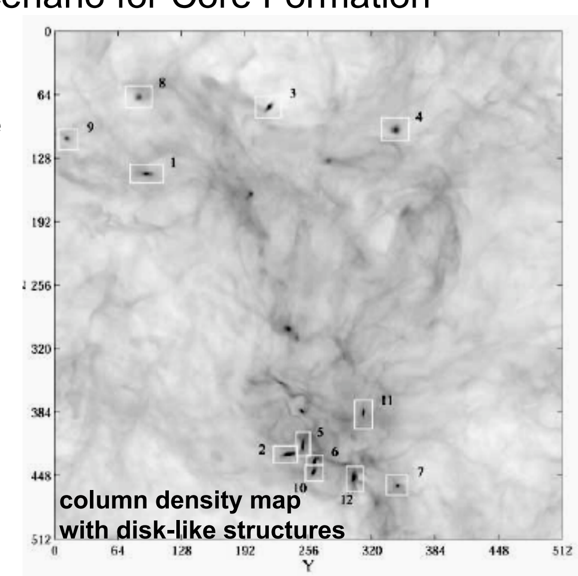
3-D MHD Simulation of Core Formation

Li, Norman, Mac-Low & Heitsch, ApJ 605 800 2004

An Alternate Scenario for Core Formation

- 512^3 element periodic box
- super critical field
- strong supersonic turbulence
- M = 64 M_J, L - 4 L_J
- followed for 3 free-fall times
- no microscopic physics

readily form prolate and tri-axial super critical cores and centrifugally supported massive disks at late times (numbered in figure)



See Crutcher et al. (ApJ 692 844 2009) for references to more recent simulations that include magnetic fields and turbulence.