# AN HEURISTIC INTRODUCTION TO RADIOASTRONOMICAL POLARIZATION

**CARL HEILES**

**Astronomy Department, UC Berkeley**

# OUTLINE

- Quantifying polarization with Stokes parameters
- Radio beats optical: We measure all Stokes parameters simultaneously
- Beam effects: squint, squash, more distant sidelobes—Arecibo and GBT
- Why is our Nature paper on DLA Zeeman splitting wrong?
- My website: a paradise of tutorials and documentation

ASTRONOMY & ASTROPHYSICS
SUPPLEMENT SERIES

MAY II **1996**, PAGE 161

*Astron. Astrophys. Suppl. Ser.* **117,** 161-165 (1996)

# Understanding radio polarimetry. III. Interpreting the IAU/IEEE definitions of the Stokes parameters

J.P. Hamaker and J.D. Bregman

Netherlands Foundation for Research in Astronomy, Postbus 2, 7990 AA Dwingeloo, The Netherlands

Received September 26; accepted October 22, 1995

**Abstract.** — In two companion papers (Paper I, Hamaker et al. 1996; Paper II, Sault et al. 1996), a new theory of radio-interferometric polarimetry and its application to the calibration of interferometer arrays are presented. To complete our study of radio polarimetry, we examine here the definition of the Stokes parameters adopted by Commission 40 of the IAU (1974) and the way this definition works out in the mathematical equations. Using the formalism of Paper I, we give a simplified derivation of the frequently-cited 'black-box' formula originally derived by Morris et al. (1964). We show that their original version is in error in the sign of Stokes $V$, the correct sign being that given by Weiler (1973) and Thompson et al. (1986).

**Key words:** methods: analytical — methods: data analysis — techniques: interferometers — techniques: polarimeters — polarization

## 1. Introduction

In a companion paper (Hamaker et al. 1996, Paper I) we have presented a theory that describes the operation of a polarimetric radio interferometer in terms of the properties of its constituent elements and in doing so unifies the heretofore disjoint realms of radio and optical polarimetry. In a second paper (Sault et al., Paper II) we apply this theory along with theorems borrowed from optical polarimetry to the problem of calibrating an interferometer array such as an aperture-synthesis telescope.

In practical applications, the theory must be supplemented by precise definitions of the coordinate frames and the Stokes parameters that are used. This problem was first addressed by the Institute of Radio Engineers in 1942; the most recent version of their definition was published in 1969 (IEEE 1969). For radio-astronomical applications, the IAU (1974) endorses the IEEE standard, supplementing it with definitions of the Cartesian coordinate frame shown in Fig. 1 and of the sign of the Stokes parameter $V$.

Most published work on actual polarimetric interferometer observations infers the source's Stokes-parameter brightness distributions from a formula derived by Morris et al. (1964). Weiler (1973) rederives their result, agreeing except for the sign of Stokes $V$. Thompson et al. (1987) include his version in their textbook, even though they suggest in their wording that they agree with Morris et al. Clearly the situation needs to be clarified; starting from a complete interpretation of the definitions, we are in a good position to do so. We shall show Weiler's version indeed to be the correct one.

*Send offprint requests to*: J.P. Hamaker, jph@nfra.nl

## 2. The Stokes parameters in a single point in the field

The definition of the Stokes parameters most frequently found in the literature is in terms of the auto- and cross-correlations of the $x$ and $y$ components of the oscillating electrical field vectors in a Cartesian frame whose $z$ axis is along the direction of propagation. Following the notation of Paper I, we represent the components of the electric field by their time-varying complex amplitudes $e_x(t), e_y(t)$. The Stokes parameters are then customarily defined by (e.g. Born & Wolf; Thompson et al. 1986):

$$\begin{aligned} I &= <|e_x|^2 + |e_y|^2> \\ Q &= <|e_x|^2 - |e_y|^2> \\ U &= 2<|e_x||e_y|\cos\delta> \\ V &= 2<|e_x||e_y|\sin\delta> \end{aligned} \quad (1)$$

Their equation (1):

$$
\begin{aligned}
I &= < |e_x|^2 + |e_y|^2 > \\
Q &= < |e_x|^2 - |e_y|^2 > \\
U &= 2 < |e_x||e_y| \cos\delta > \\
V &= 2 < |e_x||e_y| \sin\delta >
\end{aligned}
$$

(The four **STOKES PARAMETERS**). They look *awfully* complicated...

But it's not *that* complicated!

Stokes parameters are linear combinations of power measured in *orthogonal polarizations*. There are four:

$$I = E_X^2 + E_Y^2 = E_{0^\circ}^2 + E_{90^\circ}^2$$
$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$
$$U = E_{45^\circ}^2 - E_{-45^\circ}^2$$
$$V = E_{LCP}^2 - E_{RCP}^2$$

We like to write the *Stokes vector*

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} .$$

# STOKES PARAMETERS: BASICS

$$I = E_X^2 + E_Y^2 = E_{0^\circ}^2 + E_{90^\circ}^2$$

$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$

$$U = E_{45^\circ}^2 - E_{-45^\circ}^2$$

$$V = E_{LCP}^2 - E_{RCP}^2$$

The first, Stokes $I$, is total intensity. It is the sum of *any two orthogonal* polarizations[1].

The second two, Stokes $Q$ and $U$, completely specify linear polarization.

The last, Stokes $V$, completely specifies circular polarization.

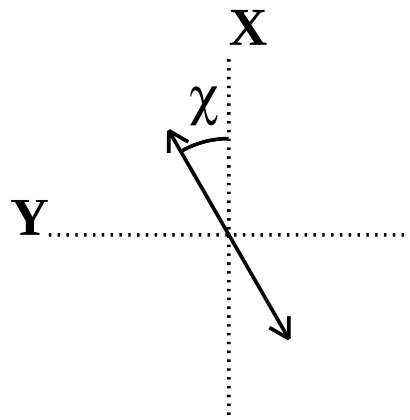
---

[1] Some ill-advised people (like at the VLA) define $I$ as the *average* instead of the *sum*. BE CAREFUL!

# CONVENTIONAL LINEAR POL PARAMETERS

$$\frac{Q}{I} = p_{QU} \cos(2\chi)$$

$$\frac{U}{I} = p_{QU} \sin(2\chi)$$




FRACTIONAL LINEAR POLARIZATION:

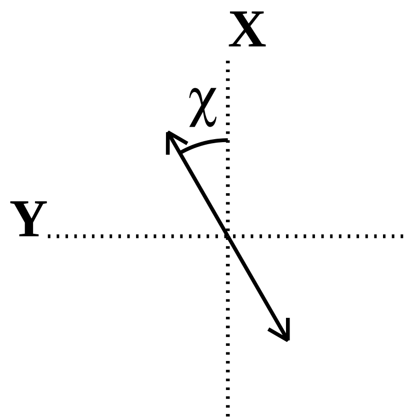
$$p_{QU} = \left[ \left( \frac{Q}{I} \right)^2 + \left( \frac{U}{I} \right)^2 \right]^{1/2}$$

POSITION ANGLE OF LINEAR POLARIZATION:

$$\chi = 0.5 \tan^{-1} \frac{U}{Q}$$

# HEY!!! LINEAR POLARIZATION "DIRECTION" ??

Look at the figure again:




THERE'S NO ARROWHEAD ON THAT "VECTOR"!! That's because it's the angle $2\chi$, not $\chi$, that's important.

Moral of this story:

- **NEVER** say "linear polarization **DIRECTION**".

- **INSTEAD**, always say "linear polarization **ORIENTATION**".

# OTHER CONVENTIONAL POLARIZATION PARAMETERS

FRACTIONAL CIRCULAR POLARIZATION:

$$p_V = \frac{V}{I}$$

TOTAL FRACTIONAL POLARIZATION:

$$p = \left[\left(\frac{Q}{I}\right)^2 + \left(\frac{U}{I}\right)^2 + \left(\frac{V}{I}\right)^2\right]^{1/2}$$

If both $p_{QU}$ and $p_V$ are nonzero, then the polarization is *elliptical*.

# THE (NON) SENSE OF CIRCULAR POLARIZATION

How is Right-hand Circular Polarization defined?

- If you're a physicist: clockwise as seen by the *receiver*.
- If you're an electrical engineer: the IEEE convention, clockwise as seen by the *transmitter*. *Hey!!! what does the receiver see???*
- If you're a radio astronomer: the technical roots are in microwave engineering, so it's the IEEE convention. Probably!! *You'd better check with your receiver engineers!* Or, to be *really* sure, *measure it yourself* by transmitting a helix from a known vantage point (and remember that $V$ changes sign when the signal reflects from a surface!).
- If you're an optical astronomer: you read it off the label of the camera and you have no idea (your main goal is the grant money, so getting the science right is too much trouble).

# THE (NON) SENSE OF STOKES $V$

OK. . . Now that we have RCP straight, how about Stokes $V$?

- If you're a physicist: $\underline{V = RCP - LCP}$.
- If you're an electrical engineer: there's no IEEE convention. Radio astronomers' convention is, historically, from Kraus (e.g. his "ANTENNAS" or his "RADIO ASTRONOMY"): $\underline{V = LCP - RCP}$. *Hey! With Kraus's definition of $V$, do physicists and engineers agree???*
- If you're an official of the International Astronomical Union (IAU): The IAU uses the IEEE convention for RCP. . . ,
  and it defines $\underline{V = RCP - LCP}$, meaning that, for $V$, the IAU *differs from both the physicist and the Kraus convention*!.

**IS ALL THIS PERFECTLY CLEAR?**

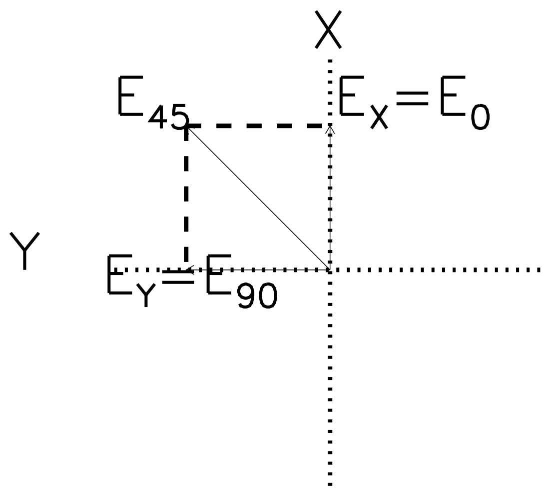
WE'RE NOT THE ONLY ONES WHO ARE CONFUSED! In his thesis, Tim Robishaw traced historical use of $V$ by astronomers in his thesis. Lets take a look:

(separate pdf file).

# REAL RADIO ASTRONOMERS MEASURE ALL STOKES PARAMETERS SIMULTANEOUSLY!

Extracting two orthogonal polarizations provides *all* the information; you can synthesize *all* other E fields from the two measured ones!

Example: Sample $(E_X, E_Y)$ and synthesize $E_{45}$ from $(E_X, E_Y)$:




To generate $E_{45}$, add $(E_X, E_Y)$ with no phase difference.

To generate $E_{LCP}$, add $(E_X, E_Y)$ with a 90° phase difference.

# CARRYING THROUGH THE ALGEBRA FOR THE TWO LINEARS . . .

It's clear that
$$E_{45^\circ} = \frac{E_{0^\circ} + E_{90^\circ}}{\sqrt{2}}$$

$$E_{-45^\circ} = \frac{E_{0^\circ} - E_{90^\circ}}{\sqrt{2}}$$

Write the two linear Stokes parameters:

$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$

$$U = E_{45^\circ}^2 - E_{-45^\circ}^2 = 2E_X E_Y$$

**STOKES $U$ IS GIVEN BY THE CROSSCORRELATION $E_X E_Y$**

To get $V$, throw a $90^\circ$ phase factor into the correlation.

# DOTTING THE I'S AND CROSSING THE T'S GIVES. . .

Carrying through the algebra and paying attention to complex conjugates and extracting the real part of the expressions yields (for sampling linear polarization $(X, Y)$:

$$I = E_X \overline{E_X} + E_Y \overline{E_Y} \equiv \mathbf{XX}$$

$$Q = E_X \overline{E_X} - E_Y \overline{E_Y} \equiv \mathbf{YY}$$

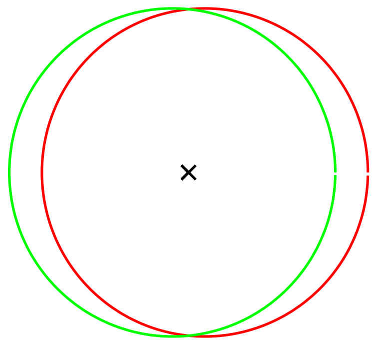
$$U = E_X \overline{E_Y} + \overline{E_X} E_Y \equiv \mathbf{XY}$$

$$iV = E_X \overline{E_Y} - \overline{E_X} E_Y \equiv \mathbf{YX}$$

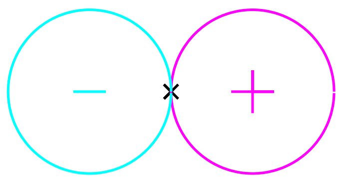
The overbar indicates the complex conjugate. These products are time averages; we have omitted the indicative $\langle\rangle$ brackets to avoid clutter.

# POLARIZED BEAM EFFECTS: BEAM SQUINT

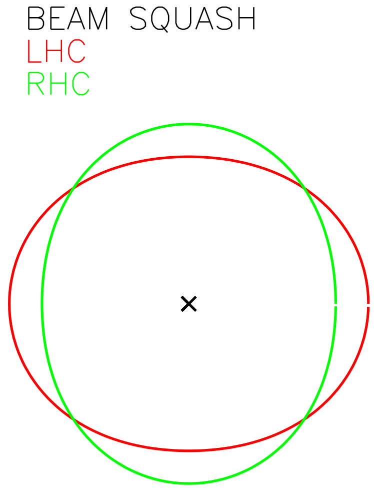
BEAM SQUINT
LHC
RHC

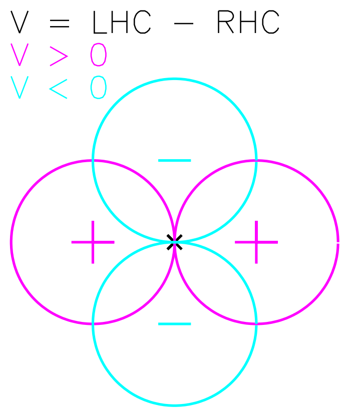


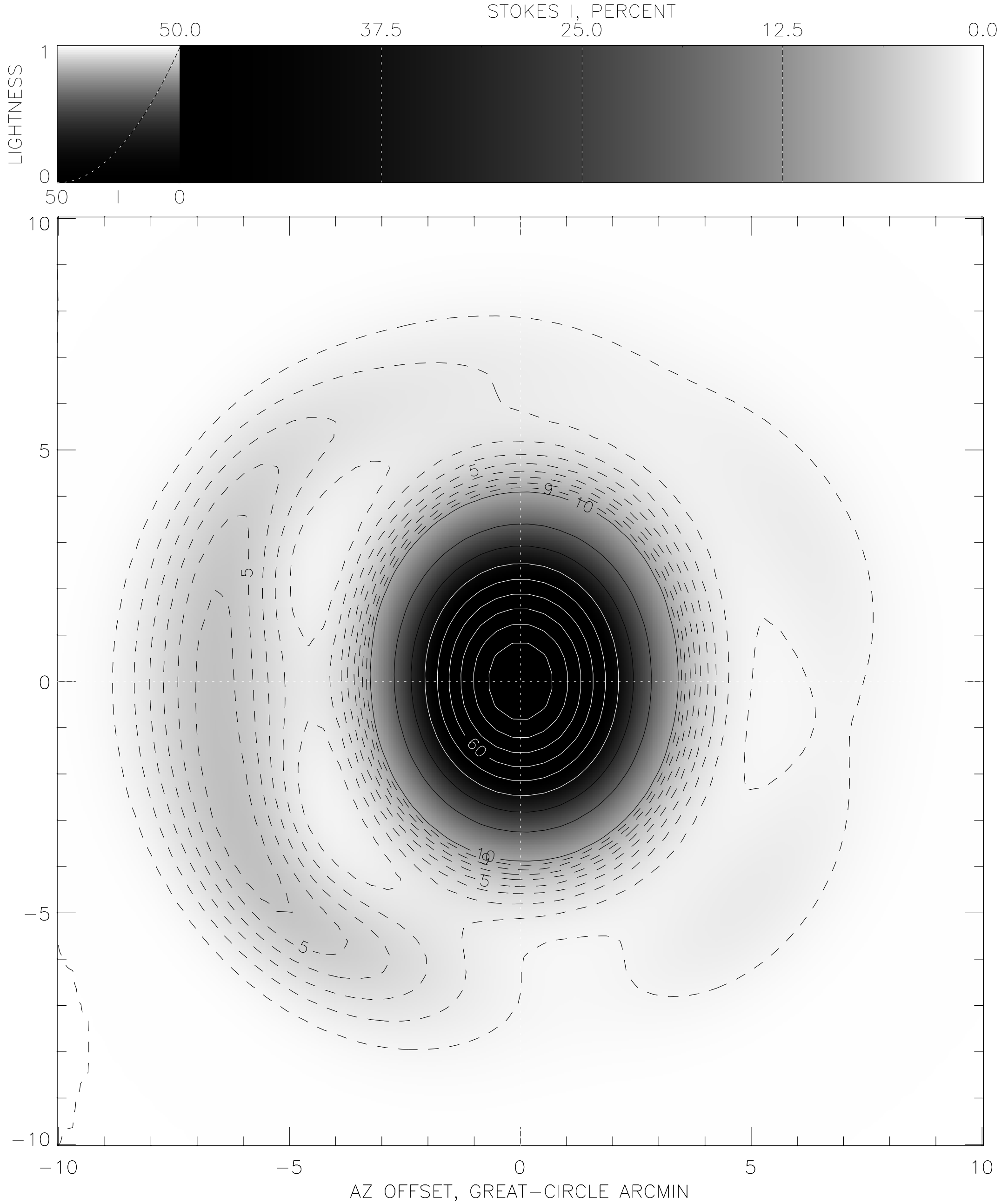
V = LHC − RHC
V > 0
V < 0

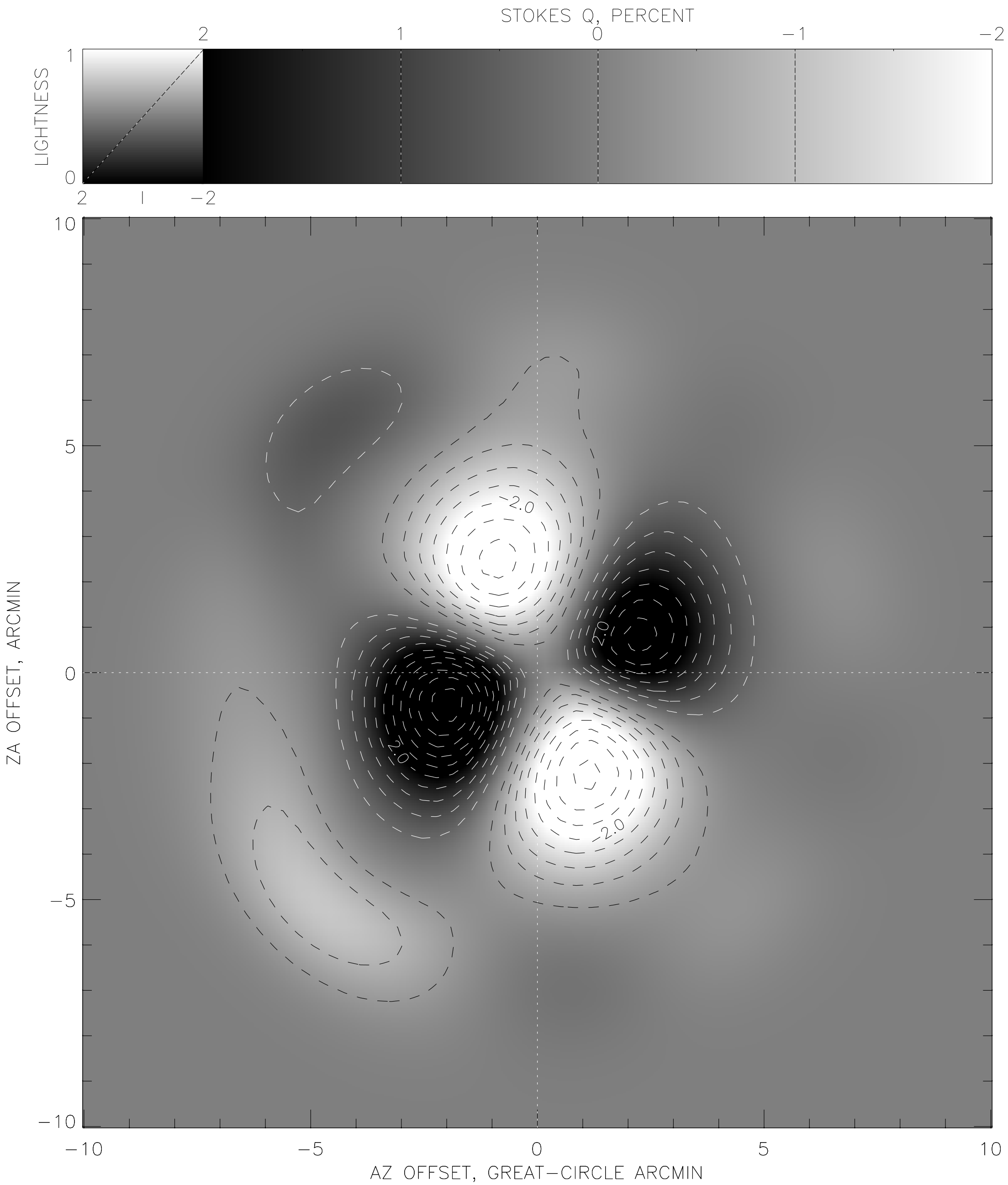


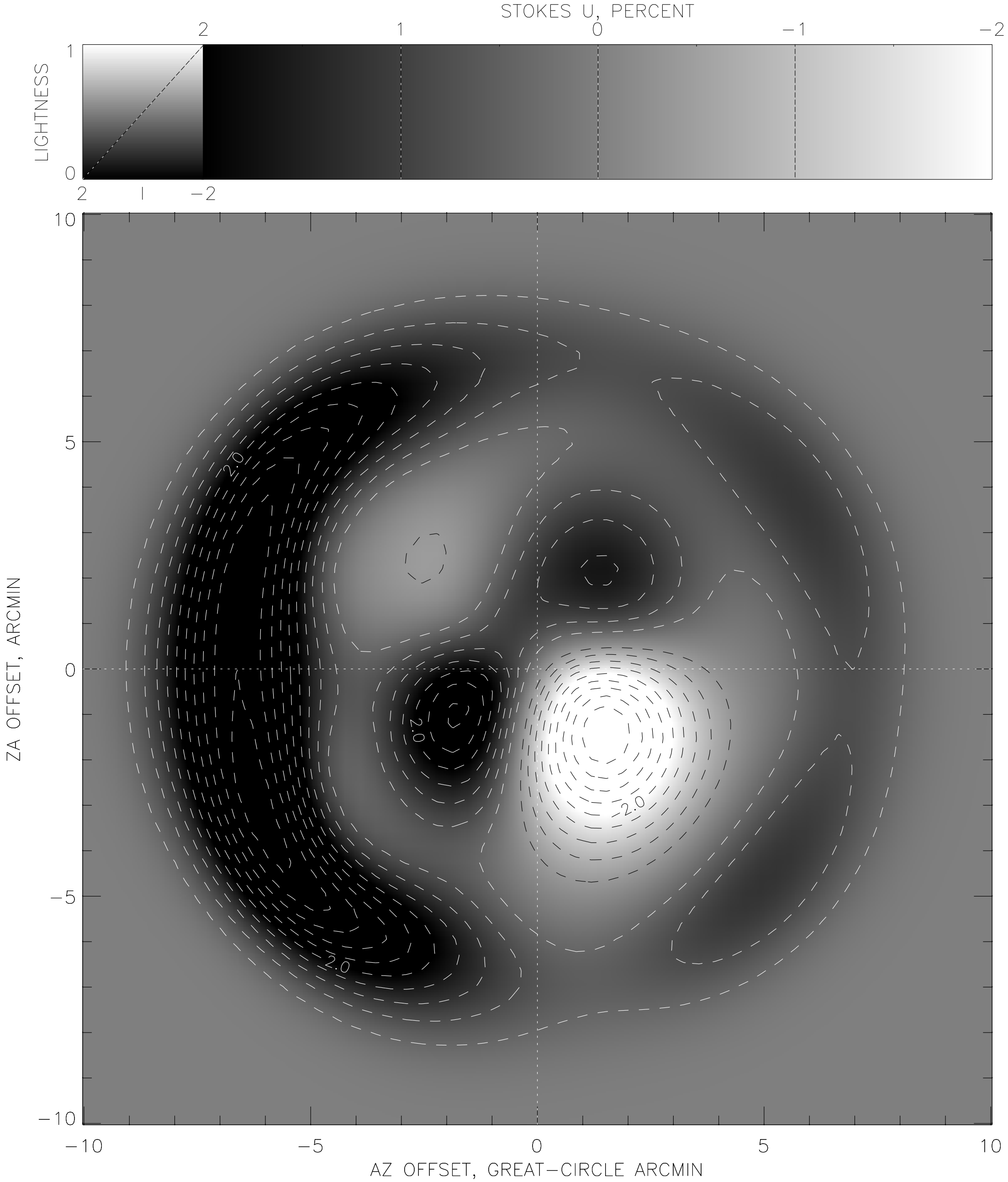
# POLARIZED BEAM EFFECTS: BEAM SQUASH

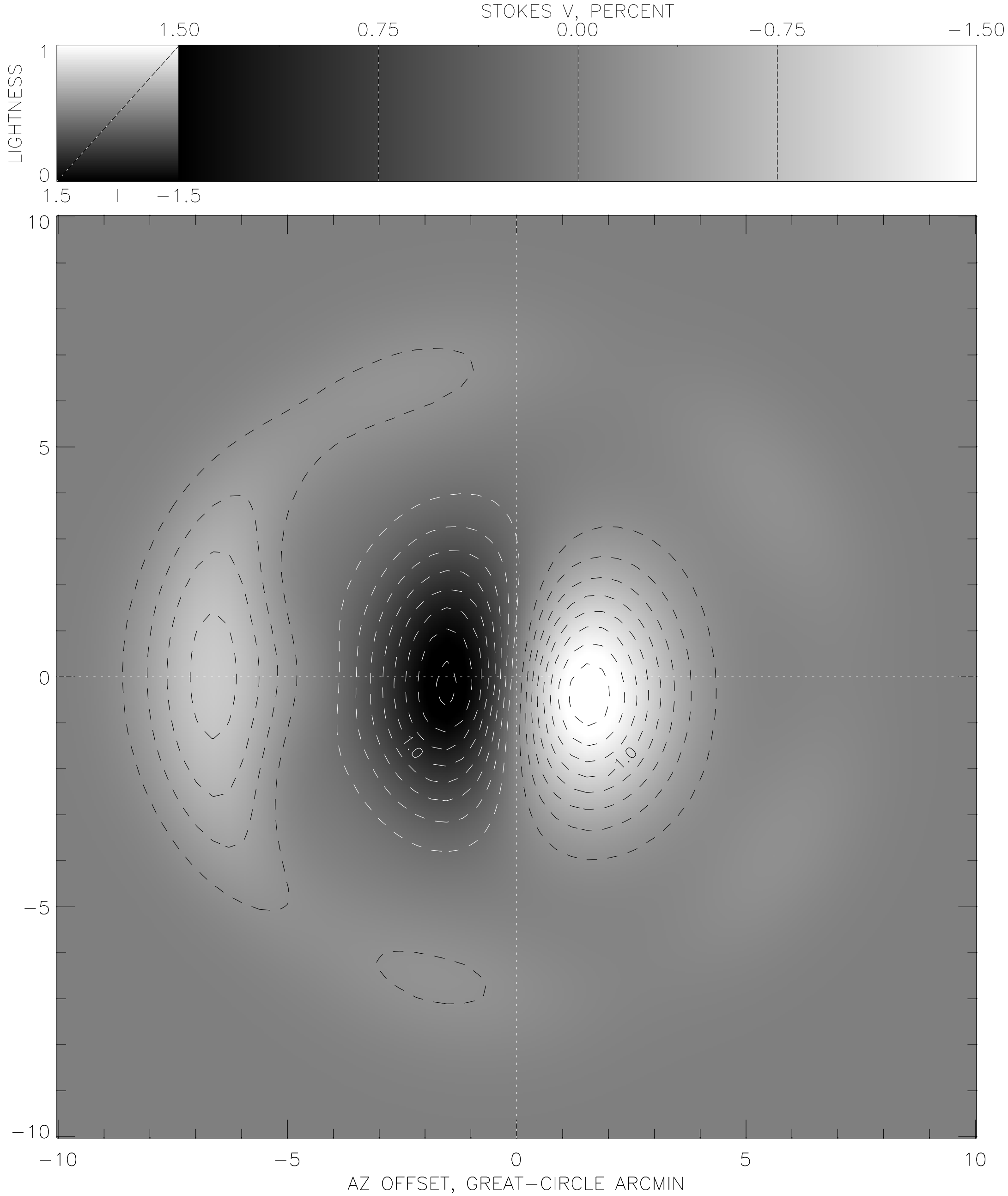


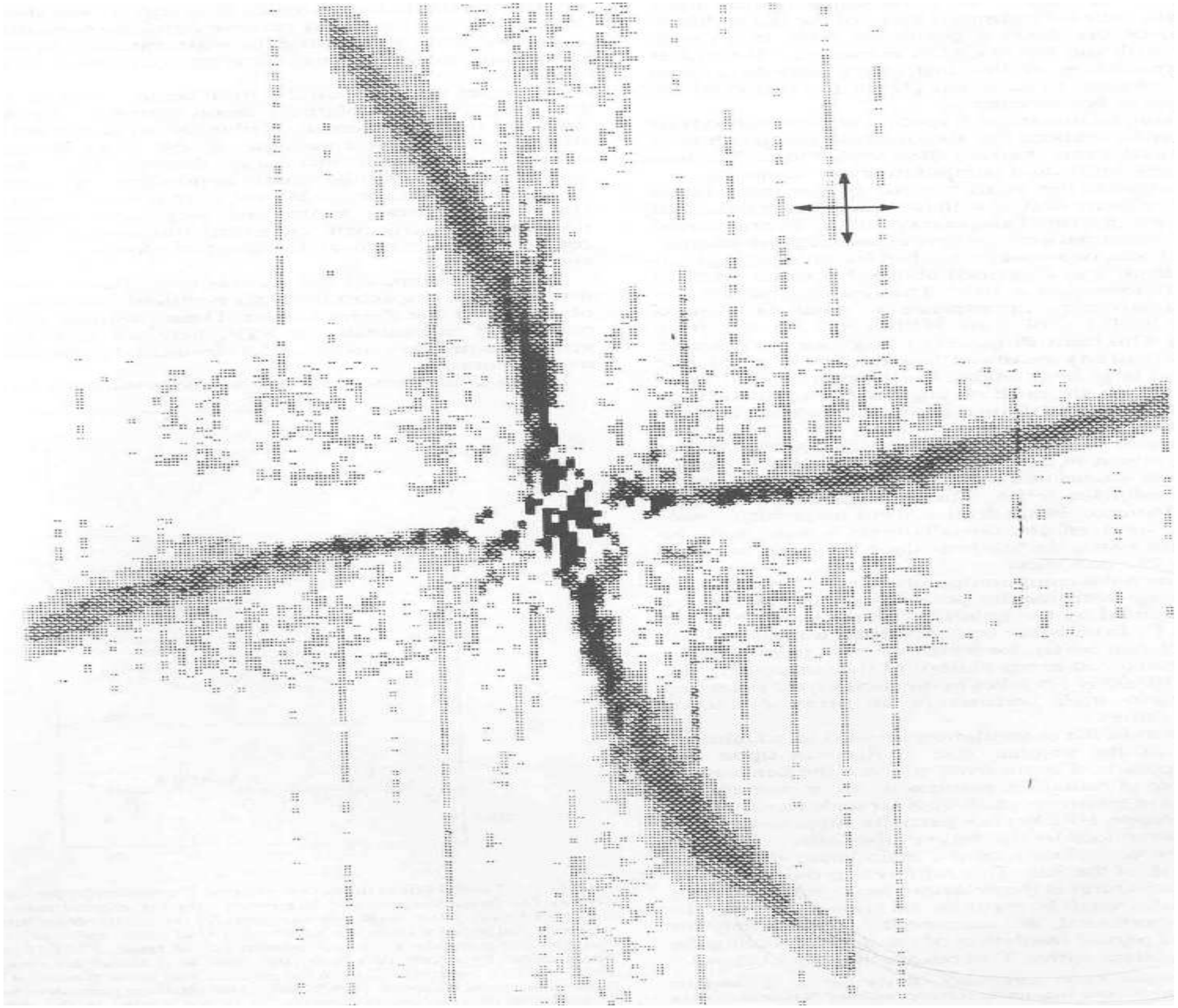

STOKES I, PERCENT
50.0
37.5
25.0
12.5
0.0
LIGHTNESS
1
0
50
I
0
ZA OFFSET, ARCMIN
AZ OFFSET, GREAT-CIRCLE ARCMIN
10
5
0
−5
−10
5
9
10
60


STOKES Q, PERCENT
2
1
0
−1
−2
LIGHTNESS
1
0
2
I
−2
10
5
0
−5
−10
ZA OFFSET, ARCMIN
−10
−5
0
5
10
AZ OFFSET, GREAT−CIRCLE ARCMIN
2.0


STOKES U, PERCENT
2
1
0
−1
−2
LIGHTNESS
1
0
2
I
−2
10
5
0
−5
−10
ZA OFFSET, ARCMIN
−10
−5
0
5
10
AZ OFFSET, GREAT−CIRCLE ARCMIN
2.0
2.0
2.0
2.0

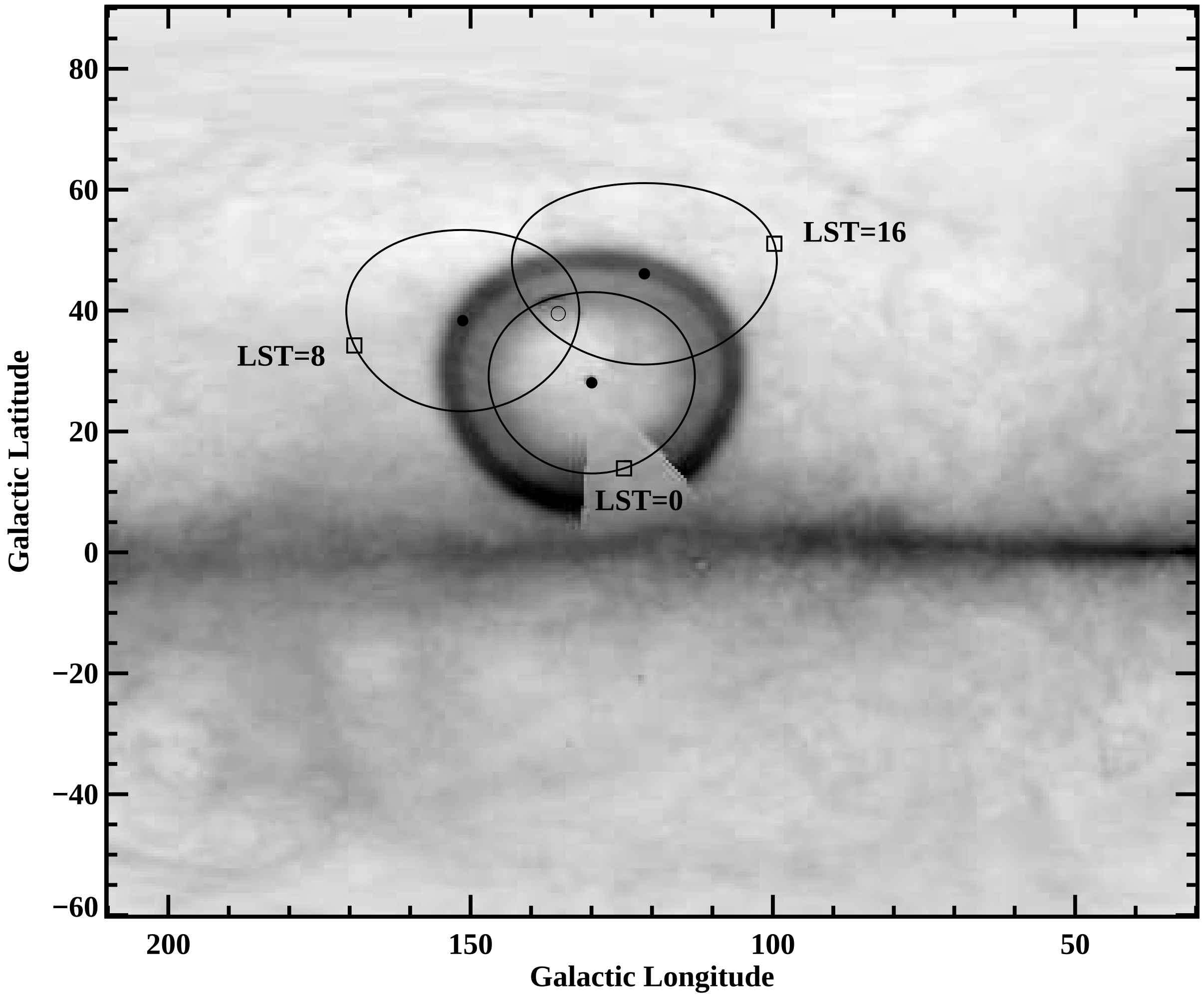

STOKES V, PERCENT
1.50
0.75
0.00
−0.75
−1.50
LIGHTNESS
1
0
1.5
I
−1.5
ZA OFFSET, ARCMIN
AZ OFFSET, GREAT−CIRCLE ARCMIN
1.0
1.0
10
5
0
−5
−10

# POLARIZED BEAM EFFECTS: DISTANT SIDELOBES



(Stokes $V$ from the Hat Creek 85-footer. Image is $120^\circ \times 120^\circ$)

Even the GBT is not sidelobe-free. Here's an approximate image of the secondary spillover in Stokes *I*—and there are also serious near-in lobes. All are *highly polarized!!!* (Robishaw & Heiles 2009, PASP, 121, 272)



# THE EFFECT ON ASTRONOMICAL POLARIZATION MEASUREMENTS

Large-scale features have spatial structure of *Stokes I*. Sidelobes in Stokes $Q$, $U$, and $V$ see this structure. The polarized beam structure interacts with the Stokes $I$ derivatives to produce **FAKE RESULTS** in the *polarized* Stokes parameters $(Q, U, V)$.

Correcting for these effects is a complicated business. First, you have to measure them; they are weak, so this is difficult. (At the GBT, Robishaw and Heiles (2009) used the Sun.) They may well be time variable, particularly at Arecibo where the telescope geometry changes as the telescope tracks. Finally, the polarized sidelobes rotate on the sky as the parallactic angle changes—and distant sidelobes might see the ground instead of the sky.

**IT'S REALLY HARD TO ACCURATELY MEASURE POLARIZATION OF EXTENDED EMISSION!!**

# WHY IS OUR NATURE PAPER WRONG?

nature

Vol 455 | 2 October 2008 | doi:10.1038/nature07264

LETTERS

## An 84-μG magnetic field in a galaxy at redshift z = 0.692

Arthur M. Wolfe[1], Regina A. Jorgenson[1], Timothy Robishaw[2], Carl Heiles[2] & Jason X. Prochaska[3]

**The magnetic field pervading our Galaxy is a crucial constituent of the interstellar medium: it mediates the dynamics of interstellar clouds, the energy density of cosmic rays, and the formation of stars[1]. The field associated with ionized interstellar gas has been determined through observations of pulsars in our Galaxy. Radio-frequency measurements of pulse dispersion and the rotation of the plane of linear polarization, that is, Faraday rotation, yield an average value for the magnetic field of $B \approx 3\,\mu$G (ref. 2). The pos-**

observations of the 21-cm absorption line show that the gas layer must extend across more than 0.03″ to explain the difference between the velocity centroids of the fringe amplitude and phase-shift spectra[9] (although the data are consistent with a magnetic field coherence length of less than 200 pc, the resulting gradient in magnetic pressure would produce velocity differences exceeding the shift of $\sim 3\,\mathrm{km\,s^{-1}}$ across 200 pc detected by very-long-baseline interferometry). By contrast, the transverse dimensions of radio beams subtended at neutral

NATURE | Vol 455 | 2 October 2008 LETTERS

715 in DLA-3C286, the magnetized gas cannot be confined by its self-gravity. Therefore, self-consistent magnetostatic configurations are ruled out unless the contribution of stars to $\Sigma$ exceeds $\sim 350 M_{\odot}\,\mathrm{pc}^{-2}$. Although this is larger than the $50 M_{\odot}\,\mathrm{pc}^{-2}$ surface density perpendicular to the solar neighbourhood, such surface densities are common in the central regions of galaxies. In fact, high surface densities of stars probably confine the highly magnetized gas

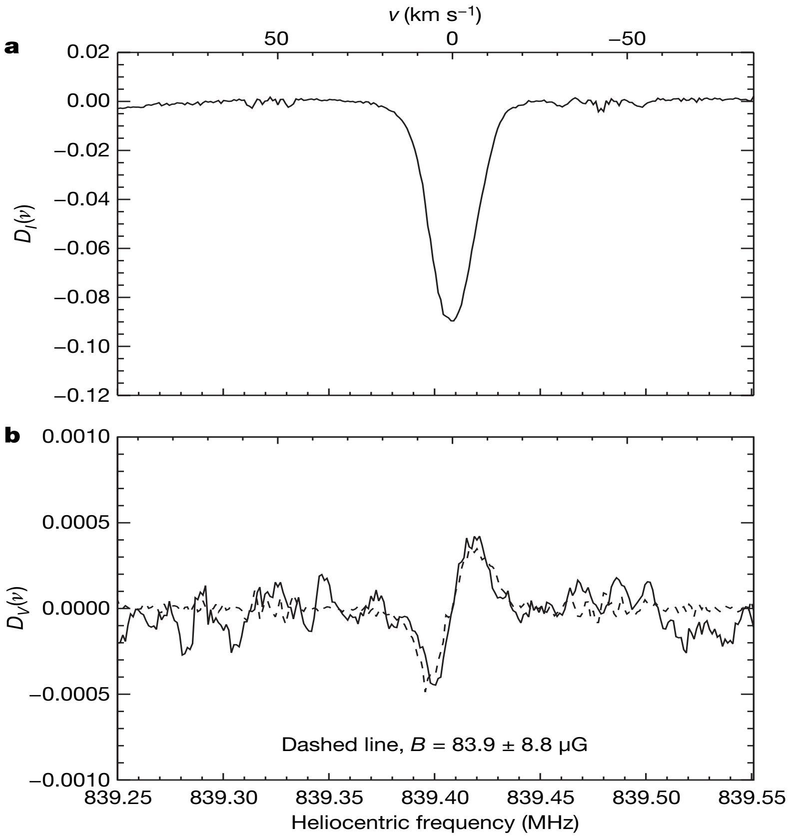



**Figure 1 | Line-depth spectra of Stokes parameters.** Data acquired in 12.6 hours of on-source integration with the GBT radio antenna. Because the GBT feeds detect only orthogonal, linearly polarized signals, whereas Zeeman splitting requires measuring circular polarization to construct $V(\nu)$,

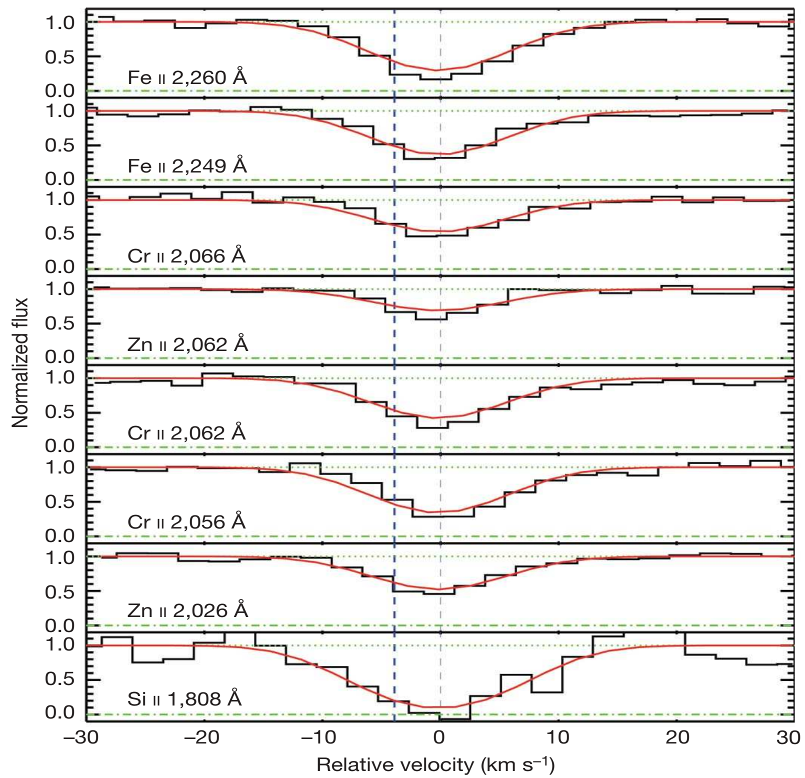



**Figure 2 | HIRES velocity profiles for dominant low-ionization states of abundant elements in the 21-cm absorber in the direction of quasar 3C 286.** Spectral resolution is $\Delta v = 7.0\,\mathrm{km\,s^{-1}}$ and the average signal-to-noise ratio per 2.1-km-s$^{-1}$ pixel is about 30:1. The bold dashed vertical line denotes the velocity centroid of the single-dish 21-cm absorption feature and the faint dashed vertical lines denotes the velocity centroid of the resonance line shown in the figure. Our least-squares fit of Voigt profiles (red) to the data (black) yields ionic column densities as well as the redshift centroid and velocity dispersion shown in Table 1 (lower and upper green horizontal lines refer to zero and unit normalized fluxes, respectively). Because refractory elements such as Fe and Cr can be depleted onto dust grains[25], we used the volatile elements Si and Zn to derive a logarithmic metal abundance with respect to solar abundances of [M/H] = −1.30. The depletion ratios [Fe/Si] and [Cr/Zn] were then used to derive a conservative upper limit on the logarithmic dust-to-gas ratio relative to Galactic values of [D/G] < −1.8.

# REMEMBER THIS # 1: AVERAGING LINEAR POLARIZATIONS!!!

Suppose you average two polarization observations together:

Observation 1 has $p = 13.6\%$ and $\chi = 2°$

Observation 2 has $p = 13.7\%$ and $\chi = 178°$

NOTE THAT THE POSITION ANGLES AGREE TO WITHIN 4 DEGREES.

If you average $p$ and $\chi$, you get $p = 13.65\%$ and $\chi = 90°$.

========= **THIS IS INCORRECT!!!!!!!!!** =========

There is only one *proper* way to combine polarizations, and that is to use the Stokes parameters. The reason is simple: because of conservation of energy, powers add and subtract.

What you must **always** do is convert the fractional polarizations and position angles to Stokes parameters, average the Stokes parameters, and convert back.

**SOME DOCUMENTATION.** You might find my website useful; it contains instructional handouts and practical IDL software.

`http://astro.berkeley.edu/~heiles/`

It has sections on (a partial list):

- *Radio Astronomical Techniques and Calibration* [specific intensity, spectral lines, polarization, characterizing the telescope beam (including "Spider scans"), LSFS (Least-Squares Frequency Switching)]
- *IDL Procedures and Instructional Handouts* [Introductory tutorial; datatypes]
- Downloading my set of IDL procedures
- *Principles of Imaging and Projections* [Four tutorials, including use of color]
- *Handouts on Numerical Analysis* [Least squares, Fourier, Wavelets, etc, etc, etc...]

In addition, we are currently working on two coherent practical writeups of "how to do polarization calibration and data analysis" for the GBT and Arecibo...