THE CO MOLECULE

1. The CO molecule has its first rotational transition ($J = 1 \rightarrow 0$) at a frequency of 115 GHz. Find its exact frequency, either from the NIST database

`http://physics.nist.gov/PhysRefData/MolSpec/Diatomic/index.html`

or the LAMBDA database (Netherlands), which has more of what you will need below

`http://home.strw.leidenuniv.nl/~moldata/`

a. Write an expression for the energy W_J of rotational level J for CO. Express the energy in units of temperature and, also, frequency in GHz. Also, write expressions for the Einstein A and the degeneracy of each J-level. You can use any data in the databases referenced above (e.g., like for the Einstein A for [J=(1-0)]). Which of these lines are not easily observable from the ground? (see , e.g.,

`http://www.submm.caltech.edu/cso/weather/atplot.shtml`).

b. Assume that the volume density is so high that the level populations are collisionally dominated. Write an expression for the fraction of molecules $f(J) = \frac{n(J)}{n(CO)}$ in state J. Make sure you include the proper statistical weight factors for the levels. *Also*, make sure that your sum over $n(J)$ gives the total number $n(CO)$—*i.e.*, make sure that you include the partition function, and properly normalize your distribution of $n(J)$.

c. For $kT \gg W_{(J=1)}$, the partition function can be approximated as an integral. Express the answer to (b) as a simple function of temperature, by evaluating the integral.

d. The optical depth of any line $\propto N_1(1 - \frac{n_2 g_1}{n_1 g_2})$, where the subscripts 1 and 2 indicate the lower and upper states. For CO, how does the optical depth of the ($J = 1 \rightarrow 0$) line depend on temperature in this high-temperature limit, assuming LTE?

How does the optical depth of a two-level system such as the CII FS IR line depend on temperature? Explain in phenomological terms why these temperature dependencies differ.

e. For levels involved in transitions below 1 THz, determine the critical densities for temperatures 10 and 30 K. You will need collisional de-excitation rates for CO by H_2. The best and easiest source is the LAMBDA database. I used the table in the 'Old CO data file', which is easier to use than the more up-tp-date one. Recall that, to determine the critical density, you need to sum over all downard collision transitions.

f. For those levels, calculate numerical values for $f(J) = \frac{n(J)}{n(CO)}$, for for $T = 10$ K and $T = 30$ K. In the optically thin limit, the line intensity $\propto$ the number in the upper state. An observer who observes the 115 GHz (or any other line, for that matter) line has to know the temperature if she wants to determine the *total* CO abundance. The CO line is rarely optically thin. However, its less-common isotopes such as ^{13}CO are commonly optically thin and used to determine CO abundance.

g. Put all the above together and make a table of J, W(J), A(J), ncrit(J), f(J) for the two

temperatures.

2. Now let's apply this to an actual example. The RADEX documentation uses as an example a uniform spherical cloud with diameter 3000 AU, T= 10K, $n(H_2) = 3000$ cm^{-3}, $f_{CO} = N(CO))/n(H_2) = 8 \times 10^{-5}$, $\Delta V(FWHM) = 1$ km/s; these give $N(H_2) = 1.35 \times 10^{20}$ cm^{-2} and $N(CO) = 1.08 \times 10^{16}$ cm^{-2}. These parameters make the radiative transfer 'interesting', with CO optical depths of a few. Let's use this example, except we will take $T = 30$K.

a. For this cloud, use your above results to calculate the optical depths of the CO lines.

b. Use the RADEX code to explore the effects of multilevel collisional excitation and radiative transfer. Since the example is 'interesting', explore the low- and high- column density cases (low and high optical depth, and thus low and high values for the escape probability β). So for the $1 \rightarrow 0$ and $2 \rightarrow 1$ lines, make a table of the outputs of the on-line RADEX output (T_x, τ, T_R) for three values of the CO column density: 100 times smaller than, equal to, and 100 times bigger than the example's $N(CO) = 1.0 \times 10^{16}$ cm^{-2}.

c. For the [J=(1-0)] line, reconcile the RADEX output line intensity T_R with the equation of transfer for a uniform cloud. *Hint: remember that the observed line is the difference between (line + continuum) and (line)*—equation (25b) in RT handout.