PROBLEM SET "all about OIII", Ay 216

1. a. Draw an energy level diagram for the three lowest terms of the O III ion. You can use Spitzer (chapter 4), Osterbrock (chapter 3), or Aller's 'Physics of Thermal Gaseous Nebulae' (p.325). Best is Osterbrock, which is the only book to have the Einstein As (as far as I am aware). In the various editions of Osterbrock, all the O-III info is in tables in chapter 3 (which exact table depends on the edition). Or, if you're courageous, dig into cloudy's files or use the NIST database.

Also, make a table showing, for each possible possible transition: the wavelength; the energy, in temperature units; the Einstein A; the collision strengths. Also, note the number of electric dipole selection rules that are violated, and which selection rules have the largest effect on the Einstein A's. Why can't you find any data for the $^1S_0 \rightarrow ^3P_0$ transition?

b. For ions with an electron configuration of p^2 or p^4 like O III, the ground term is a ^{3}P with three closely-spaced levels. Using the fact that the collision strength of a level within a term is proportional to its statistical weight (equation 3.22 of Osterbrock or 3.21 of Osterbrock & Ferland), prove Osterbrock's statement following equation 3.22, 'the rate of collisional excitation...to the ^{1}D and ^{1}S levels is very nearly independent of the distribution of ions among the $^3P_0$, $^3P_1$, and $^3P_2$ levels.'

c. For the low-density limit, calculate the total number of photons emitted per cm^3 per sec in the $^1D_2 \rightarrow ^3P$ transition (includes all levels in the ^{3}P term). Express your answer in terms of the electron density. The density of O itself should not appear in your answer; instead, use the abundance of O III to hydrogen, which you may take equal to the volume density ratio of O III to electrons. Leave the temperature dependence in explicitly.

d. Calculate the total number of photons emitted per cm^3 per sec in the Hα line. Assume that all electrons come from H.

e. Find the 'standard' cosmic abundance ratio—i.e., the ratio of the number of nuclei—of O to H from Spitzer, Table 1.1.[1] Assume that depletion is equal to that for ζ Oph. Assume that all the O is O III. For what temperature is the number of photons in the $^1D_2 \rightarrow ^3P$ O III transition equal to that in Hα? (This is why H II regions sometimes look green in color photographs).

2. The object of this problem is to show that that the intensity of optical O III line ($^1D_3 \rightarrow ^3P$) *decreases* as the oxygen abundance *increases* (thus illustrating the difficulty of determining elemental abundances from optical spectra!). The effect occurs because O III is a primary coolant in H II

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[1] In principle, You should use a more modern reference. Good ones include Osterbrock's book (better: the new Ferland/Osterbrock edition); Table 4 of the Savage/Sembach review (1996 ARAA, 34, 279); and best, what's buried in Gary Ferland's CLOUDY code. But for our purposese, does it really matter? Spitzer's table has two entries, one showing typical depletion; Savage/Sembach have three entries (B stars, HII region, Sun). And then there's the uncertainties in transition probabilities (in this day and age, yet!); peruse Table 2 in Savage/Sembach for some sobering comparisons. All this impacts on the basic uncertainties in gas phase ISM abundances. . .

regions. For O III, two types of cooling transitions are important: the IR transitions among the three ^{3}P levels in the ground term, and the optical transitions between the ^{3}P ground term and the $^1D_2$ state. Because O III is the major coolant, the H II region temperature decreases with increasing O III abundance; because the optical transitions are so temperature sensitive, they get weaker because of the lower temperature.

a. Calculate the approximate critical density for the IR transitions. Since this is a three-level problem, you ought to do it properly; see equation (3.31) of Osterbrock.

b. Write an approximate equation for the O III cooling rate, including both the optical and IR transitions. Assume the low density limit for both the optical lines and the IR transitions, just to make things easier. Note that your result from (a) implies that this isn't always the case: for fairly compact H II regions the high-density limit is more appropriate for the IR lines, while for H II regions visible in external galaxies the low density limit is likely to apply.

c. Write an approximate equation for the heating rate in terms of the H II region gas temperature, the electron density, and the temperature of the exciting star (see class notes).

d. Put all of the above together and calculate the equilibrium temperature of the H II region. Include the O/H ratio as a free parameter. Evaluate the temperature for the *un*depleted value of O/H from Spitzer's table 1.1 (to make the temperature come out right—we have neglected too many coolants!), assuming one of Spitzer's O5 stars heats the H II region.

e. Show that the *total* derivative of the optical line intensity with respect to the O/H abundance (don't forget the contribution from the partial derivative of the cooling with respect to gas temperature!) is negative, thus accomplishing the objective of the problem. You can do the derivative numerically if you wish.