PROBLEM SET IRDUST: IR FROM GRAINS IN AN HII REGION

Suppose we have an O5 star with Spitzer's Table 5.3 UV photon output of $N_u = 5 \times 10^{49}$ s^{-1}, $T_* = 47000$ K, $L \approx 10^{5.9} L_\odot = 3.0 \times 10^{39}$ erg s^{-1} lighting up an HII region of uniform density $n_e = 10^3$ cm^{-3}. Suppose further than the dust/gas ratio is typical, i.e. that the mass of heavy elements residing in grains is 0.01 times that of the Hydrogen, and that the grain mass density (i.e., the mass density of an individual grain) is 3 gm cm^{-3}.

Further suppose, for ease of calculation, that the grain size distribution is the MRN distribution with PDF $P(a_r) \propto a_r^{-3.5}$, where $a_r = \frac{a}{a_{min}}$, for a between 200 A and 2000 A (a smaller a_{min} of about 50 A is more realistic, of course, and then there are the VSGs and PAHs, which we are neglecting).

1. Determine the actual size distribution $\frac{dn}{da_r}$, where dn is the number of grains per cm^3 in size range da_r. That is, find $\mathcal{K}$ in the expression $\frac{dn}{da_r} = n_H \mathcal{K}\ a_r^{-3.5}$, where n_H is the volume density of H nuclei.

2. From the whole HII region, what is the total IR luminosity from grain emission (equal to the total heating rate of the grains) from absorbing Lα photons? Remember that a well-defined fraction of the recombinations produce Lyα photons that are trapped, heating the dust, and that the remainder don't heat the dust (why?).

3. From the whole HII region, what is the total IR luminosity from grain emission (equal to the total heating rate of the grains) from absorbing starlight below 912 A? *Hint:* Check first to see if the dust's absorption optical depth is large. If it is, then you can assume that none of the star's photons make it out of the HII region. Does more grain IR come from Lyα or starlight?

4. From the whole HII region, what is the total emission in all cooling lines from the HII gas? How does this compare with the IR from grain emission?

5. One could determine the spectral shape of the grain IR emission by finding the temperature of grains versus a, multiplying by the effective radiating area, and multiplying by the appropriate $B_\nu(T)$. This is a tedious process. So instead, use the size distribution you derived above in (**1**) to pick a single representative grain size to represent all grains. Find the grain temperature and locate the approximate location of the peak of the IR emission.

To make things straightforward, I suggest assuming that $Q_{abs} = \frac{2\pi a}{\lambda}$ for $\frac{2\pi a}{\lambda} < 1$ and $Q_{abs} = 1$ for $\frac{2\pi a}{\lambda} > 1$. (This is why we use $a_{min} = 200$A). [Note: According to Tielens book, the dielectric constant factor goes as $\frac{\lambda_0}{\lambda}$ for most materials. If you wish, you can incorporate this, but you'll have to pursue his book to find a suitable value of λ_0.]

What is a "representtative grain size"? Suppose that the radiative power input for a single grain $\propto a^x$. Then the total power absorbed by all grains of that size $\propto a^x P(a)$. A representative grain has size $\langle a^x \rangle^{1/x}$, where $\langle a^x \rangle = \left(\frac{\int a^x P(a) da}{\int P(a) da} \right)$. One final point: you need to adjust the number of grains with the representative size so that the grain mass with respect to the H mass retains its proper ratio of 0.01.