ESSENTIALS OF WEEK 1, ASTRONOMY 127

1. Introduction to ISM:

dust clouds visible on photographs imply presence of interstellar dust, hence gas.

spectral lines in absorption tell column density, ionization state, temperature, velocity. Temperatures range from 10 K to $> 10^6$ K.

spectral lines in emission allow mapping of the interstellar gas that emits. To emit photons of energy $h\nu$, the gas must be hot enough with $kT \gtrsim h\nu$.

To emit in the optical, temperatures must be of order 10^4 K. HII regions are a prime example.

We will cover HII regions in detail. Their physics is straightforward because the heating and ionization source is well-defined: the central star.

Physics of ISM is non-LTE, as opposed to situation in terrestrial laboratory which is usually LTE.

2. Review of LTE:

Boltzmann distribution says $n(E)dE \propto g(E)\exp(-E/kT)dE$. $g(E)$ is the "statistical weight" or "degeneracy". Temperature T is a measure of the average energy, with a multiplicative coefficient depending on the energy dependence of $g(E)$.

For a gas, the one-dimensional velocity distribution is $n(v_x)dv_x \propto \exp(-mv_x^2/2kT)dv_x$. The energy is $mv^2/2$, and the energy distribution is $n(E)dE \propto E^{1/2}\exp(-E/kT)dE$. In this case, $g(E) \propto E^{1/2}$. With $g(E) \propto E^{1/2}$, the average energy is $3kT/2$.

Quantum systems, with discrete levels labelled j, say, have a statistical weight that depends on the quantum numbers of the levels. For example, the H-atom has $E_j - E_{j=1} = E_0(j^{-2} - 1)$ and $g_j = 2j^2$.

3. Non LTE:

Consider a two level system. Achieve *statistical equilibrium* by balancing the rates per unit volume up and down between the two levels. Let n_2 be the number per cm^3 in state 2 and R_{21} be the probability of a transition between 2 and 1 in one second. Then the total rate down per cm^3 is n_2R_{21}. Statistical equilibrium is the condition

$$n_2R_{21} = n_1R_{12} \tag{1}$$

Processes are collisional and radiative, so we can write in complete generality

$$n_2(R_{21}^{coll} + R_{21}^{rad}) = n_1(R_{12}^{coll} + R_{12}^{rad}) \tag{2}$$

If collisions dominate radiation by a large factor, *and* if the gas velocity distribution is Maxwellian at temperature T_k (the *kinetic temperature*)—i.e., if the gas kinetic energy distribution is a Boltzmann—then n_2/n_1 is also a Boltzmann at the same temperature T_k:

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \exp(-E_{21}/kT_k) \tag{3}$$

This is known as the "principle of detailed balancing", and arises from fundamental considerations in thermodynamics and statistical mechanics. This fact allows us to write

$$\frac{R_{12}(T_k)}{R_{21}(T_k)} = \frac{g_2}{g_1} \exp(-E_{21}/kT_k) \tag{4}$$

which is a relation between the upwards and downwards collision rates. These collision rates are, fundamentally, atomic properties, and this is a relation between these atomic properties. Thus, this relationship holds under *any* circumstances, LTE or non-LTE, and whether or not collisions dominate.

Collisional rates are expressible in terms of the usual kinetic theory manner, with a thermal velocity v_{th} and a cross section σ. For an atom or ion of some kind (the *collidee*) that is being hit by some other particle (the *collider*), the probability that the collidee suffers a collision in one second (assume downward collisions for sake of definiteness) is

$$R_{21} = n_{collider} \langle \sigma_{21} v_{th} \rangle \tag{5}$$

and, of course,

$$R_{12} = \frac{g_2}{g_1} n_{collider} \langle \sigma_{21} v_{th} \rangle \exp(-E_{21}/kT_k) \tag{6}$$

Note that the above are rates of collision *per collidee*. To get the rates *per unit volume*, you must multiply by $n_{collidee}$—so that for example, the rate of downward collisions per unit volume is

$$n_2 R_{21} = n_2 n_{collider} \langle \sigma_{21} v_{th} \rangle \tag{7}$$

We consider three types of collisions:

(1): neutral-neutral collisions. Cross sections are typically "geometrical", the area of the electron cloud—order of a Bohr orbit.

(2): electron-ion collisions. The electrostatic attraction brings the particles closer together, increasing the effective cross section, by an amount that increases with decreasing velocity. Thus the effective cross section depends on velocity. Roughly, $\sigma \propto v^{-2}$, i.e. $\sigma \propto T^{-1}$ and $\sigma v \propto T^{-1/2}$. These collisional rates can be written

$$R_{21}^{el-ion} = n_e \frac{8.6 \times 10^{-8} \Omega}{g_2 T_4^{1/2}} \quad \mathrm{sec}^{-1} \tag{8}$$

where T_4 is temperature in units of 10^4 K and Ω is the "collision strength", which depends on the quantum mechanical details and is typically of order unity. As might be expected, this rate corresponds to a cross section that is somewhat larger than that of a Bohr orbit.

(3): electron-proton recombination. This process, with an electron hitting a bare proton to make a hydrogen atom plus a photon, is characterized by a very low cross section, of order 10^{-6} times that of the above processes. Roughly, the recombination rate per proton is

$$R_{e-p\ recombination} = n_e 2.7 \times 10^{-13} T_4^{-0.7} \mathrm{sec}^{-1} \tag{9}$$