University of California, Berkeley
Physics 221b Spring 2004

Problem Set 1
due Feb. 6, 2004

1. In class we discussed the creation and annihilation operators $a_i^\dagger$ and a_i and their transformations under a change of basis (change of operators),

$$a_i^\dagger = \sum_j b_j^\dagger c_{ji} \qquad (1)$$

where the matrix c_{ji} is unitary, $\sum_j c_{jk}^* c_{ji} = \delta_{ik}$. We used the claim that

$$\sum_i a_i a_i^\dagger = \sum_j b_j b_j^\dagger \ . \qquad (2)$$

Use the above relations to show this is true.

2. Consider 3 identical particles, each which can be in state a or b.
   - What is the probability for these particles to all be in the same state classically?
   - How about if they are quantum mechanical bosons?
   - For bosons, what is the normalized state in terms of kets of one particle states for the case where two of the particles are in state a and one is in state b? (i.e. in terms of $|a\rangle$ and $|b\rangle$?)
   - For bosons, what is the normalized state in the Fock space for the case where two of the particles are in state a and one is in state b? (i.e. in terms of $a_a^\dagger$ and $a_b^\dagger$ and the Fock vacuum $|0>$?)

3. Consider three identical fermionic particles in 3 different states i, j, k. What is the normalized state in terms of kets of one particle states for this system?

4. Merzbacher problem 21.8.

Please show your work so that the grader can give you credit for effort even if you get the wrong answer. Also, please try to make your work readable!