University of California, Berkeley
Physics 221b Spring 2004

Problem Set 10
due Monday April 26, 2004

1. Merzbacher exercise 23.14.

   (a) Show that the total probability for emitting from a prescribed classical current distribution $\vec{j}(r,t)$ a net total number of n photons into a specified set R of photon modes, and none into any other mode, is given by a Poisson distribution

   $$P_n(R) = e^{-\bar{n}}\frac{r^n}{n!} \tag{1}$$

   (I.e., there is a set of modes $k_1, k_2, \ldots k_m$ and we require photons only going into those modes and adding up to a total of n photons.) Here, $\bar{n}$ is the sum of the mean number of photons emitted into all modes. Calculate what r and $\bar{n}$ correspond to for a general current.

2. Consider the forced harmonic oscillator we discussed in class:

   $$H = \omega(a^\dagger a + f^* a + f a^\dagger) \tag{2}$$

   Consider a transformation of the form

   $$\bar{H} = e^S H e^{-S} \tag{3}$$

   and take $S = \lambda_1 a + \lambda_2 a^\dagger$.

   (a) What does the requirement

   $$\bar{H}^\dagger = \bar{H} \tag{4}$$

   impose on λ_1, λ_2? Use this to eliminate λ_2.

   (b) Using Merzbacher chapter 3 identities for $e^A B e^{-A}$ or some other source, expand out $\bar{H}$ as a power series in H and commutators with S, and show that the expansion truncates. (This should be in terms of λ_1.)

(c) Require $\bar{H}$ to be of the form $\alpha a^\dagger a + C$ to find both λ_1 and C.

3. Consider the operators

$$L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu) \tag{5}$$

(a) What is the algebra satisfied by these operators? I.e. what is

$$[L_{\mu\nu}, L_{\rho\sigma}]\ ? \tag{6}$$

The right hand side should be constants and combinations of $L_{\mu\nu}$. You can assume that $\partial_\mu x^\nu = \delta_\mu^\nu$.

(b) What are the symmetries of the right hand side of the commutation relations under the exchange $\mu \leftrightarrow \nu$, $\sigma \leftrightarrow \rho$ and $\mu\nu \leftrightarrow \rho\sigma$ (i.e. $\mu \leftrightarrow \rho$ and $\nu \leftrightarrow \sigma$ simultaneously)? Explain why this is what you expect given the left hand side.

(c) Consider an antisymmetric tensor $\epsilon^{\rho\sigma}$. What is

$$\frac{i}{2}\epsilon^{\rho\sigma} L_{\rho\sigma} x^\mu\ ? \tag{7}$$

(d) Consider instead the matrix

$$(S_{\rho\sigma})_\mu^{\ \nu} = i(g_{\rho\mu} g_\sigma^\nu - g_{\sigma\mu} g_\rho^\nu) \tag{8}$$

and a vector V^μ. What is

$$(S_{\rho\sigma})_\mu^\nu V_\nu\ ? \tag{9}$$

(e) (*optional*) and find its commutation relations. In the above, the μ and ν refer to the components of the matrix. Compare to those above for $L_{\rho\sigma}$.

Please show your work so that the grader can give you credit for effort even if you get the wrong answer. Also, please try to make your work readable!