University of California, Berkeley
Physics 221b Spring 2004

**Problem Set 12**
**due May 10, 2004**

Note: This will not be returned before the final on May 13, probably, so please make a copy if you want one for yourself for the take-home.

1. In class we discussed conserved currents associated with symmetries. Consider a free scalar field Lagrangian,

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi \tag{1}$$

and consider the effect of multiplying the field $\phi$ by a phase:

$$\phi'(x) = e^{i\alpha}\phi(x) \tag{2}$$

and the complex conjugate for $\phi^\dagger$, where $\alpha$ is a real constant.

   (a) Show that this is a symmetry of the Lagrangian.

   (b) Find the associated conserved current (classical, i.e. don't worry about the ordering of the fields).

   (c) Now consider the fermionic Lagrangian:

$$\bar{\psi}(i\partial\!\!\!/ - m)\psi \tag{3}$$

   and the chiral phase transformation

$$\begin{aligned} \psi &\to e^{i\alpha\gamma^5}\psi \\ \psi^\dagger &\to \psi^\dagger e^{-i\alpha\gamma^5} \end{aligned} \tag{4}$$

   and show that this is a symmetry of the Lagrangian only when the mass $m = 0$.

   (d) Find the conserved current when $m = 0$ for the chiral phase transformation above.

2. Consider the following (nonrenormalizable) interacting Fermi theory of two types of fermions, $\psi$ and $\tilde{\psi}$:

$$\mathcal{L} = \bar{\tilde{\psi}}(i\partial\!\!\!/ - m)\tilde{\psi} + \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\bar{\psi}\psi\bar{\tilde{\psi}}\tilde{\psi} \tag{5}$$

(a) What would be wrong with having the interaction be

$$g\psi^\dagger\psi\tilde{\psi}^\dagger\tilde{\psi} \tag{6}$$

instead?

(b) Write down in terms of the fields the first order matrix element where a "tilde" particle and "tilde" anti-particle of momentum $\vec{p}$ and $\vec{q}$ annihilate into an electron and anti electron, of momentum $\vec{p}'$ and $\vec{q}'$ respectively.

(c) In the interaction, expand the fields out in operators, use $\tilde{c}, \tilde{d}^\dagger$ etc for the $\tilde{\psi}$, etc.

(d) Use the requirements on the initial and final states to reduce the sum to only those operators which are non-zero between the initial and final states. Choose the initial and final particles to have some definite (but unspecified) spins.

(e) Do the integral over $x$ to get something of the form

$$(2\pi)^4\delta^{(4)}(p'+q'-p-q)\left(\frac{m}{Vp_0}\right)^{1/2}\left(\frac{m}{Vq_0}\right)^{1/2}\left(\frac{m}{Vp'_0}\right)^{1/2}\left(\frac{m}{Vq'_0}\right)^{1/2}\mathcal{M} \tag{7}$$

(f) To get the rate, we need $|\mathcal{M}|^2$. Now assume that the initial spins are unpolarized and the final spin are not observed, i.e. sum $|\mathcal{M}|^2$ over the final spins and average over the initial spins (i.e. sum over all of them and divide by 1/4). Express your result in terms of the coupling, the masses of the particles and the initial momenta of the tilde particles. (You can use the conservation of momentum that you know is multiplying the entire expression to do this.)

(g) How does your result change if the two initial particles have mass $\tilde{m}$ rather than $m$ and $\tilde{m} < m$? What is the constraint on the initial particle 3-momenta in the center of mass frame for the process to happen?

Please show your work so that the grader can give you credit for effort even if you get the wrong answer. Also, please try to make your work readable!