University of California, Berkeley
Physics 221b Spring 2004

Problem Set 2
due Feb. 13, 2004

1. Five identical spin 0 particles are situated at the corners of a pentagon (all sides equal length). Define the z-axis to go through the center and in the direction normal to the plane of the triangle. The whole system is free to rotate about the z-axis. Using statistics considerations, obtain restrictions on the magnetic quantum numbers corresponding to J_z. (As a starting point note that under a rotation around z the wavefunction for this system will transform as $e^{iJ_z\theta}$).

2. Two unpolarized electrons are scattered. How does the cross section at angle $\pi/2$ compare to that for two distinguishable identical particles at angle $\pi/2$?

3. Use Young tableaux to do the following: How many irreducible SU(5) representations result when you combine 3 particles (each having 5 possible values)? What are the dimensions and symmetry (totally symmetric, totally antisymmetric, mixed) properties of each? Show that the total number of states in the resultant configurations adds up to the original number of states of the 3 particles.

4. Show that for two particles of total spin s each that the ratio of the number of symmetric states to antisymmetric states is $(s+1)/s$.

5. (Merzbacher problem 21.5). In the second quantized formalism, define the additive position and total momentum operators

$$\begin{aligned} \mathbf{r}_{op} &= \int a^\dagger(\mathbf{r})\, \mathbf{r} a(\mathbf{r}) d^3r \\ \mathbf{p}_{op} &= \int a^\dagger(\mathbf{p})\, \mathbf{p} a(\mathbf{p}) d^3p \end{aligned} \tag{1}$$

where

$$a(\mathbf{p}) = \int \langle \mathbf{p}|\mathbf{r}\rangle a(\mathbf{r}) d^3r \ . \tag{2}$$

We are taking $\mathbf{p}$ continuous here and neglecting spin. Show that the commutator for bosons is

$$[\mathbf{r}_{op}, \mathbf{p}_{op}] = i\hbar N \mathbf{1} \tag{3}$$

where N is the operator representing the total number of particles. (Hint: The algebra can be made simpler if you note that $[a(\mathbf{p}), a^\dagger(\mathbf{r})] = \langle \mathbf{p}|\mathbf{r}\rangle$, which can be seen from the definitions.) Also, to simplify the calculation you might want to just consider the commutator for different components of $\mathbf{r}_{op}, \mathbf{p}_{op}$, e.g. $r_{op,i}$ and $p_{op,j}$ for arbitrary i, j.

Derive the Heisenberg uncertainty relation for position and momentum of a system of bosons and interpret the result.

6. Merzbacher problem 21.7 (not exercise, problem).

Please show your work so that the grader can give you credit for effort even if you get the wrong answer. Also, please try to make your work readable!