University of California, Berkeley
Physics 221b Spring 2004

Problem Set 5
due March 5, 2004

Note that "problem" means "problem" not "exercise"! If you think you'll need your homework set before the midterm, please make a copy, as they may not be graded by then.

Problem 1 has been corrected (had an extra sum), for problem 5, N electrons are in the well (N particles).

1. We considered spin waves in class, and used that the sum over couplings

$$\phi(\kappa) \equiv \sum_j J_{j\ell} e^{i\kappa(\ell - j)} \tag{1}$$

is independent of ℓ.

(a) Show that this is true (hint:take $\ell \to \ell + 5$ for example). You can use that $J_{j\ell} = J_{j+q\, \ell+q}$ for any shift q and that the sum over j is infinite (neglect edge effects). So for instance, we have

$$\sum_j J_{j\ell} = \sum_j J_{j+q\, \ell+q} = \sum_{j+q} J_{j+q\, \ell+q} = \sum_j J_{j\ell+q} \tag{2}$$

where we used that in summing over all values j we are also summing over all values $j+q$. This shows that $\sum_j J_{j\ell}$ is independent of ℓ.

(b) Show that $\phi(\kappa)$ is an even function of κ. To do this you might need to use both the symmetry of $J_{j\ell}$ and the translation invariance.

2. We derived a spin wave energy

$$E(\kappa) = 2\hbar^2 s(\sum_j J_{j\ell}) - 2\hbar^2 s\phi(\kappa) + B\hbar \tag{3}$$

in class. What is this energy (or dispersion relation, as someone correctly noted) when the spins have only nearest-neighbor couplings, with value J?

3. Merzbacher *problem* 22.1.

4. In class we discussed the Thomas Fermi model using an equation relating the potential and number density based on interactions in atoms. Consider another case where the potential is given to you exactly,

$$V(r) = \alpha r \tag{4}$$

In the Thomas Fermi approximation, what is the extent of the electron density if there are N electrons in the well? Ignore any inter-electron interactions. Give the answer in terms of N, α, and the electron mass m.

Please show your work so that the grader can give you credit for effort even if you get the wrong answer. Also, please try to make your work readable!