University of California, Berkeley
Physics H7A Spring 2002

Problem Set 3
due Feb. 15, 2002
typos hopefully corrected!

Reading assignment: Kleppner and Kolenkow, section 2.5, Note 2.1

1. In class, we discussed why we approximate the gravitational force on the surface of the earth with $g = 9.8ms^{-2}$, even though the gravitational force on an object of mass m is more precisely $-GM_{earth}m/r^2$. We wrote, for something a distance h above the earth's surface:

$$\begin{aligned} F &= \frac{-GM_{earth}m}{r^2} = -\frac{GM_{earth}m}{(R_{earth}+h)^2} \\ &\sim -\frac{GM_{earth}m}{(R_{earth})^2}\left[1 - 2(\frac{h}{R_{earth}}) + 3(\frac{h}{R_{earth}})^2 + \cdots\right] \end{aligned} \tag{1}$$

   To leading order we just use $g = GM_{earth}/R^2_{earth}$.

   - What is the dependence on height to leading order in the height? (Thanks for the requests to clarify the earlier wording!)
   - Take $M_{earth} = 6.0 \times 10^{24} kg$, $R_{earth} = 6.4 \times 10^3 km$ and $G = 6.7 \times 10^{-11} Nm^2/kg^2$. Say we go somewhere at high altitude, e.g. Los Alamos, which has an altitude of 2.2 km above sea level. What is the first order correction to g numerically, ignoring the extra mass between you and the center of the earth at Los Alamos? What is the ratio between the correction to g and g itself?
   - What is the gravitational force between two people (say each weighs 50 kg) who are standing 1 m apart in free space? How does this compare to the force on one of them when on the earth's surface, from the earth(ie what is the ratio)?

2. Put a black hole at the origin of the coordinate system, with mass M. Put two masses, each with mass m, a distance R away from the black hole, separated by a very small angle θ, ie the distance between them is approximately $R \sin\theta \sim R\theta \equiv y$ (Taylor again). What is the difference

(direction and magnitude) between the gravitational forces acting on these two masses, due to the mass M, to leading order in y? What is the effect on the two masses, or say an object which extends between them?

By leading order in a variable I mean, take a function $f(y)$ and expand it in y, e.g. via a taylor expansion, $f(y) = a + by + cy^2 + ...$, where a, b, c are constants. The leading order term in y is the term by, unless $b = 0$, in which case it is cy^2, unless $b = c = 0$, in which case it is the next term etc etc.

3. K & K 2.22

4. Using the system in K & K 2.22, draw the force diagram on the center piece of rope. Replace the tension T by some other force $-C$, where $|C| = |T|$. Replace g by $-g$. What can you say about the forces on this piece of "rope" now, what is their sum? *This is a very straightforward problem, if it seems easy it is because it is!*

   What you've just done is the operations which make this system work in another really interesting context, discovered also by Hooke. To solve the above system you used that the tension on each bit of string cancels the other forces acting on it, both horizontally and vertically.

   If you replace the tension forces (which have each bit of string pulling on the string next to it) by compression forces (i.e. something *pushing* on the elements next to it), this effectively changes the sign of the forces on each element from its neighbors. That is, if you replace the rope by a bunch of stone or something that pushes rather than pulls like string, then you reverse the forces that each element exerts. In the horizontal direction this is fine, as you just need to make sure that the external trees can support the push from the stone rather than the pull from the string. In the vertical direction however, gravity still acts by pushing down. The way to have the forces cancel when tension (pulling) is replaced by compression (pushing) is to reverse the sign of gravity on the system, i.e. turn it upside down. That is, make the downward curve an upward curve–an arch. So this is one way to see what is involved when an arch is stable.

5. K & K 2.7 (look at 2.8 for a hint)

6. K & K 2.26
7. K & K 2.31
8. K & K 2.34