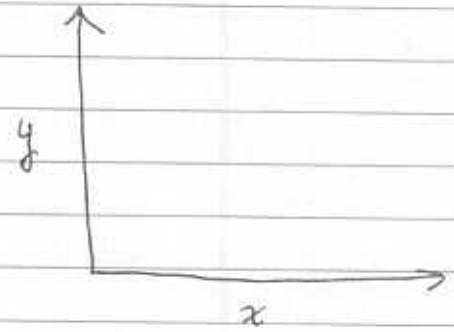
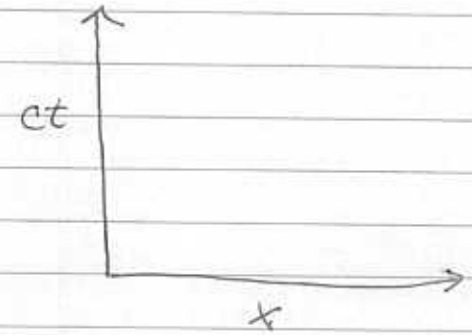
Spacetime Geometry
2/25/03

Usual Euclidean Space

"Spacetime" = Minkowski space





z not shown

distance^2: $\Delta x^2 + \Delta y^2 + \Delta z^2$

two points (x_1, y_1, z_1) & (x_2, y_2, z_2)

$$dist^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

y, z not shown

distance^2: $c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$

two pts (t_1, x_1, y_1, z_1) & (t_2, x_2, y_2, z_2)

$$\Delta s^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2$$

Why is distance special?

Euclidean:
same if you rotate coords

Spacetime:
same if you go to coord system w/ velocity v:
"rotate" by angle as done below.

"Rotate"

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

"Go velocity $v \to$ angle α"

$$x = x'\cosh\alpha + ct'\sinh\alpha$$
$$ct = x'\sinh\alpha + ct'\cosh\alpha$$

where $\cosh\alpha = \gamma$

$\sinh\alpha = \frac{v}{c}\gamma$

$\tanh\alpha = v/c$

Euclid | Spacetime

Rotate Systems
Consider system
w/ primed coords.

Where is (new) x' axis?

x' axis:
$x' = x_0'$, $y' = 0$

x' axis
$x' = x_0'$, $ct' = 0$

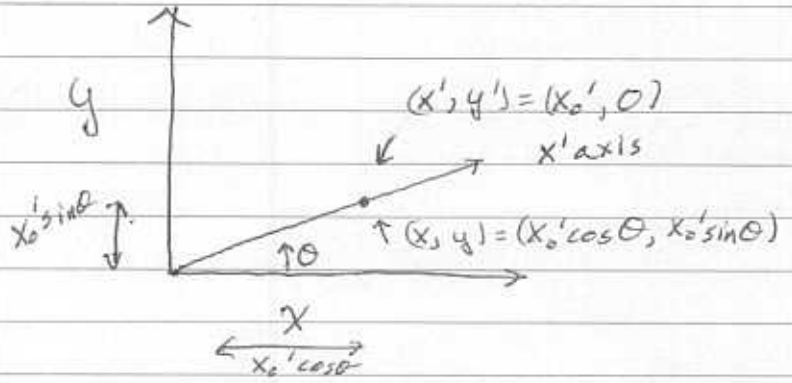
in terms of old
unprimed coords

$$x = x'\cos\theta - y'\sin\theta$$
$$= x_0'\cos\theta$$
$$y = x'\sin\theta + y'\cos\theta$$
$$= x_0'\sin\theta$$

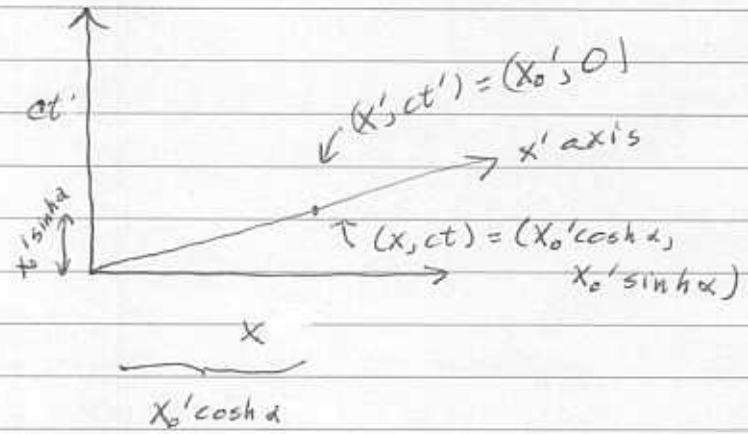
to get axis
Change x_0', plot

$$x = x'\cosh\alpha + ct'\sinh\alpha$$
$$= x_0'\cosh\alpha$$
$$ct = x'\sinh\alpha + ct'\cosh\alpha$$
$$= x_0'\sinh\alpha$$

to get axis,
Change x_0', plot



on x' axis, $\tan\theta = y/x$ coords of any point on x' axis



on x' axis, $\tan\theta = \dfrac{ct}{x}$ coords of any point on x' axis.

so $\tan\theta = \tanh\alpha$.

Where is (new)
y' or ct'
axis?

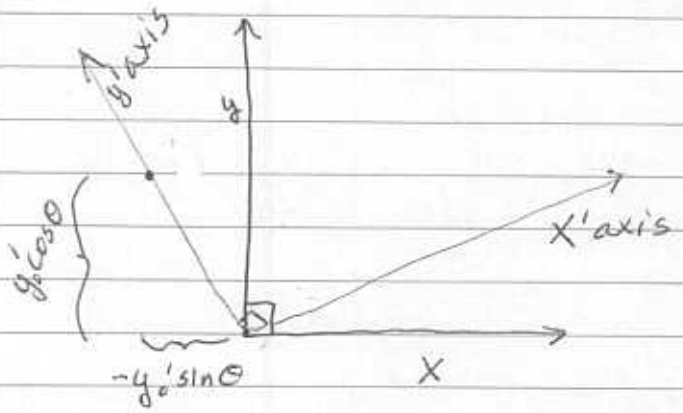
y' axis
$x'=0, y'=y_0'$

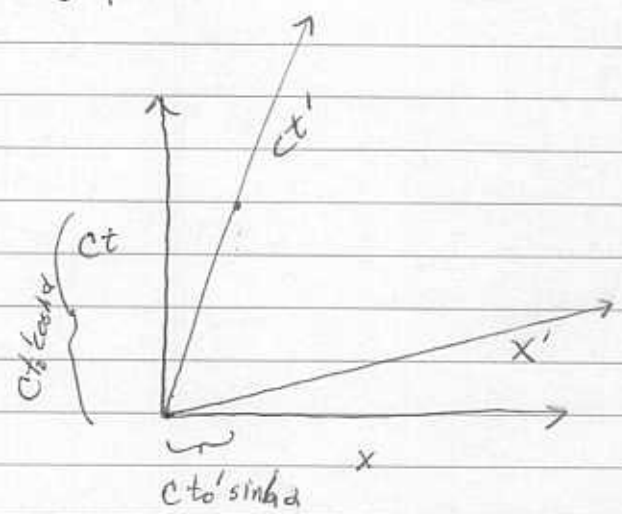
$$x = x'\cos\theta - y'\sin\theta$$
$$= -y_0'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$
$$= y_0'\cos\theta$$

ct' axis
$x'=0, \; ct'=ct_0'$

$$x = x'\cosh\alpha + ct'\sinh\alpha$$
$$= ct_0'\sinh\alpha$$
$$ct = x'\sinh\alpha + ct'\cosh\alpha$$
$$= ct_0'\cosh\alpha$$

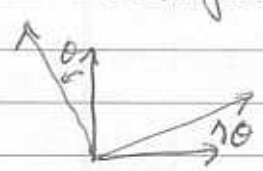
Change y_0', ct_0', plot.

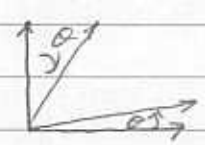




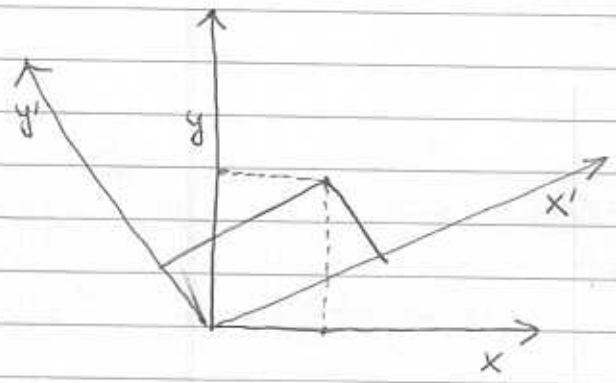
To measure coords
in either system:
draw lines parallel
to both coord
axes & see
where they intersect axes.

angles between all pairs of axes: θ ($\alpha - \theta$)

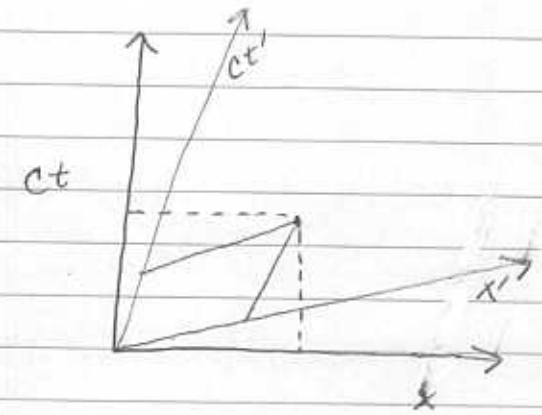




Euclid



Spacetime



Coords are where lines hit axes:

dotted lines: x, y coord (x_0, y_0)
solid lines: x', y' coord. (x_0', y_0')

dotted: x, ct coord (x_0, ct_0)
solid: x', ct' coord (x_0', ct_0')

distances same:
eg distance^2 from origin of point

$$d^2 = (x_0 - 0)^2 + (y_0 - 0)^2$$
$$= (x_0' - 0)^2 + (y_0' - 0)^2$$

can calculate using formulae for x, y in terms of x', y'.

"distances" same
eg distance^2 from origin

$$\Delta s^2 = c^2(t_0 - 0)^2 - (x_0 - 0)^2$$
$$= c^2(t_0' - 0)^2 + (x_0' - 0)^2$$

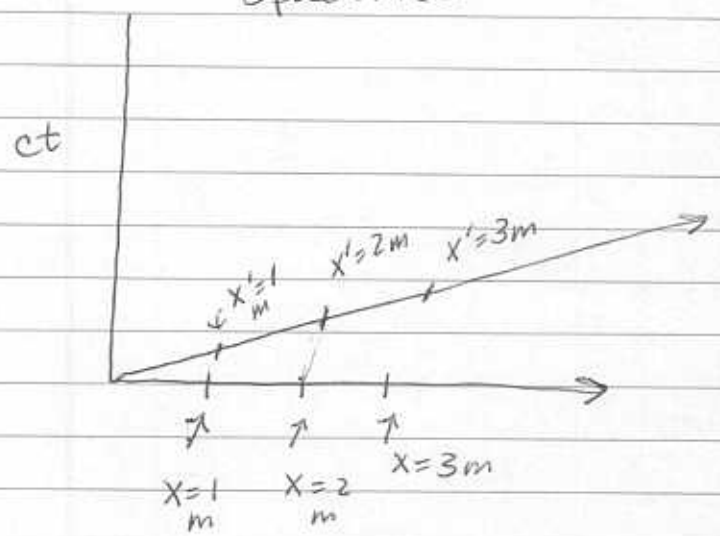
can calculate using formulae for x, ct in terms of x', ct'

Some new things about spacetime:

- light rays always at $ct' = x'$ & $ct = x$ (45°)
- ct' & x' don't <u>look</u> orthogonal (ie linearly indep).
  they are though.
  "distance" is different, so is notion of this.
  paper is Euclidean.

- spacing on x, x' axis different.

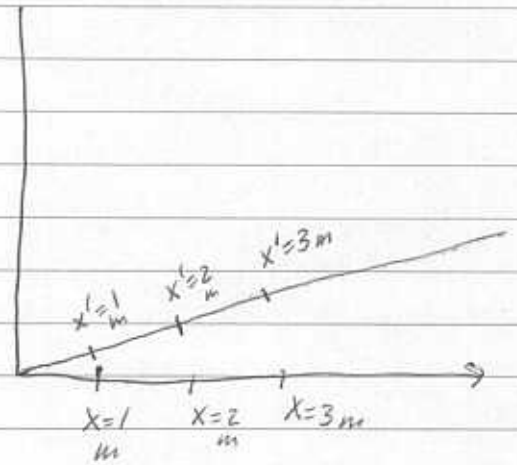
Spactime



get spacing from "lengths" preserved.

From origin:
"length²" $c^2t'^2 - x'^2$
$= ct^2 - x^2$
preserved.

vs. Euclid



From origin
length² $x'^2 + y'^2$
$= x^2 + y^2$
preserved.