National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Electric Field Conjugation-based wavefront correction algorithm for high contrast imaging systems

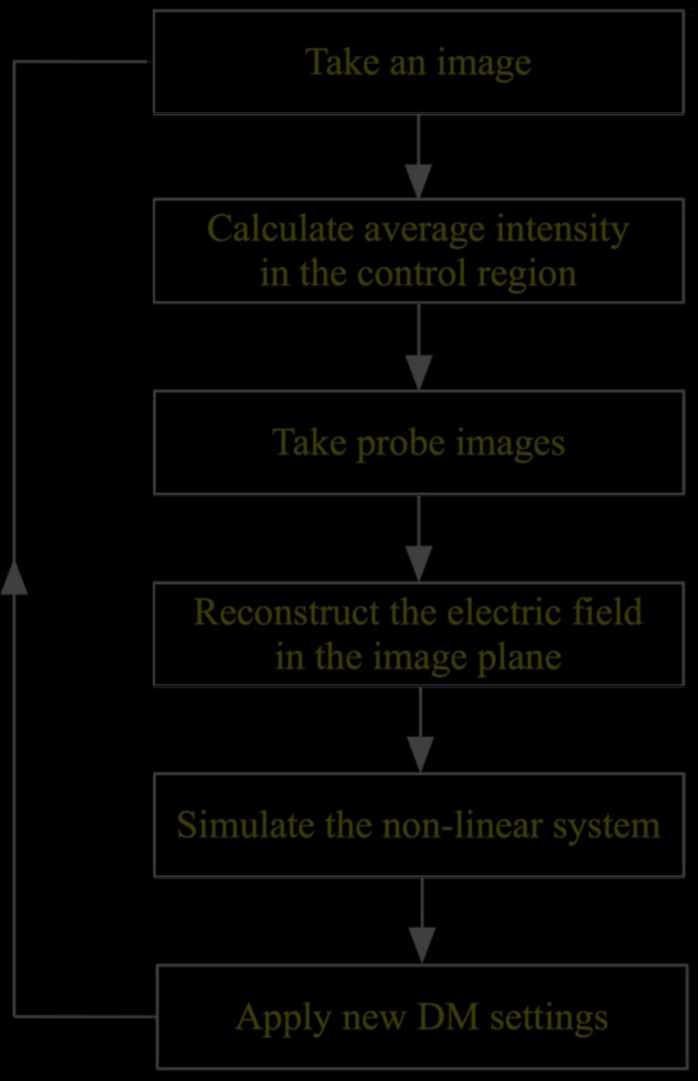
Experimental results

Amir Give'on[1]
Brian Kern[1]
Stuart Shaklan[1]
Dwight Moody[1]
Rus Belikov[2]

[1] Jet Propulsion Laboratory
[2] Princeton University

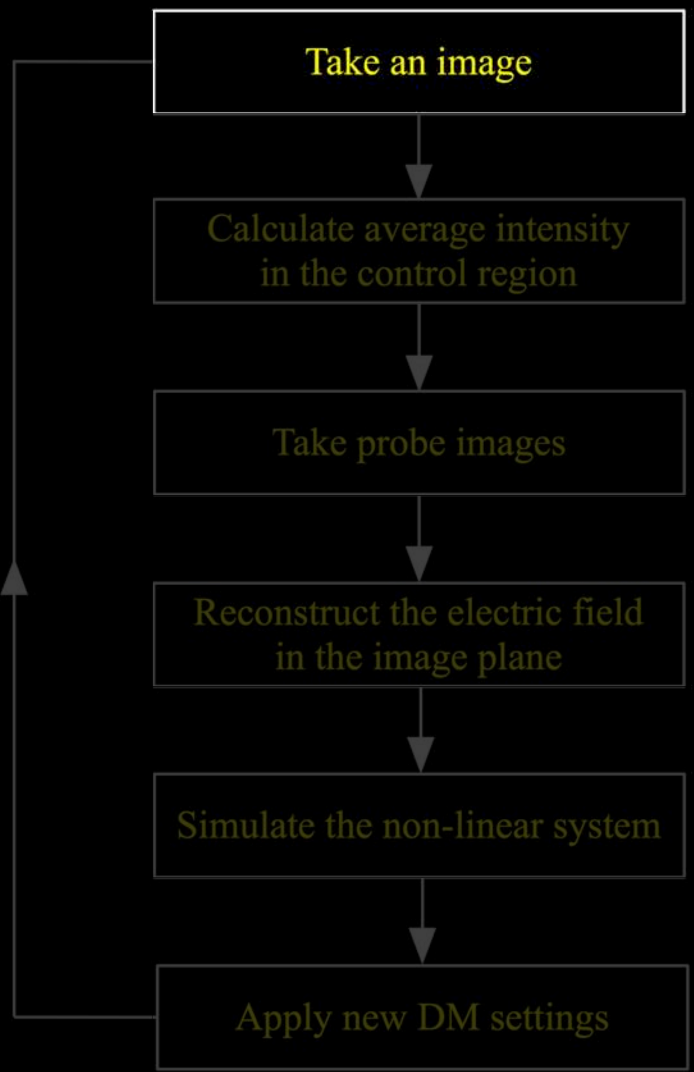
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The algorithm

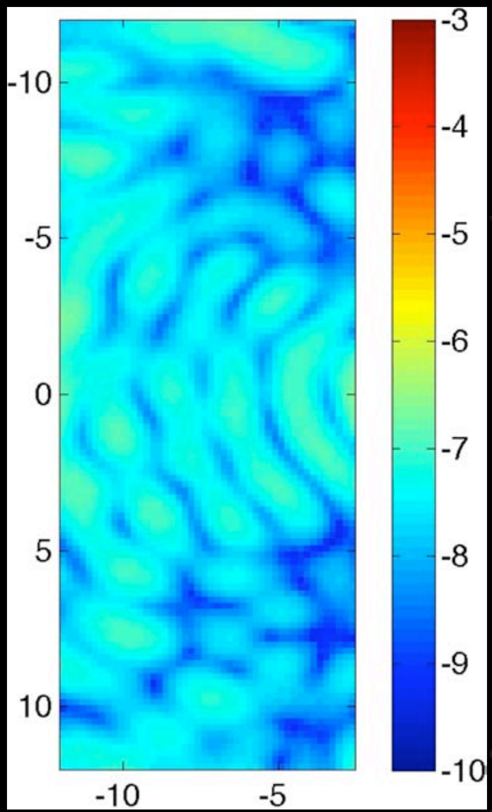


National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The algorithm



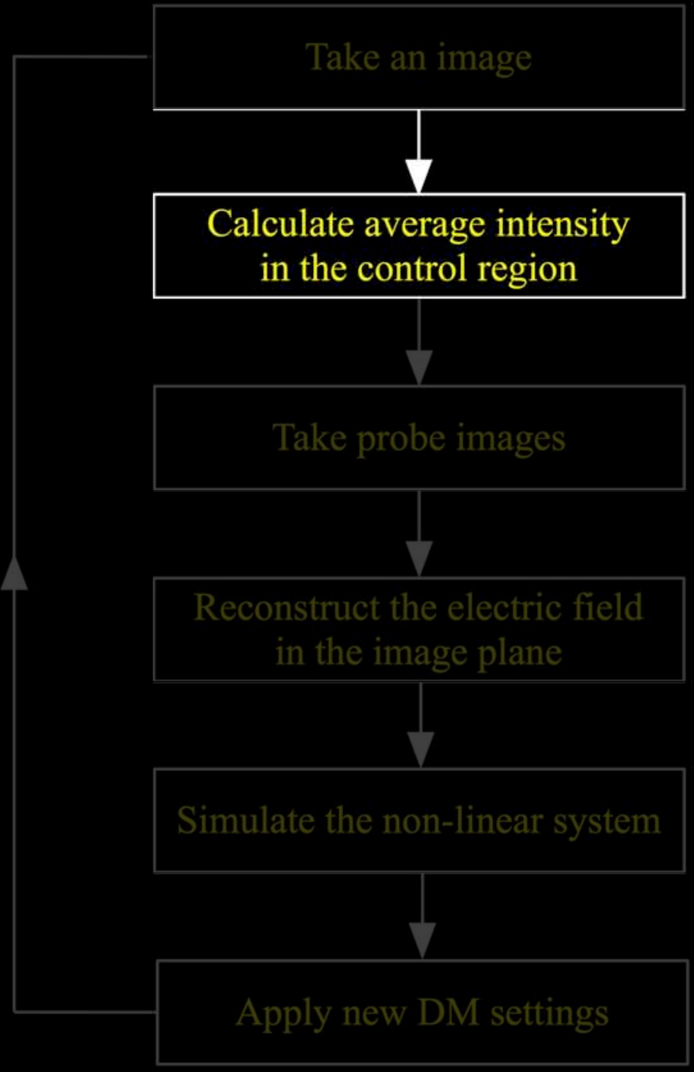
Measured image



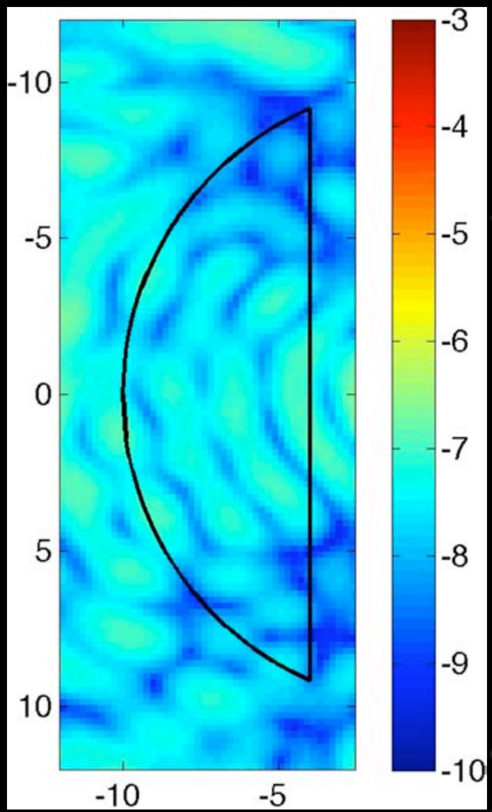
Measurements were taken at 2% around 800nm

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The algorithm



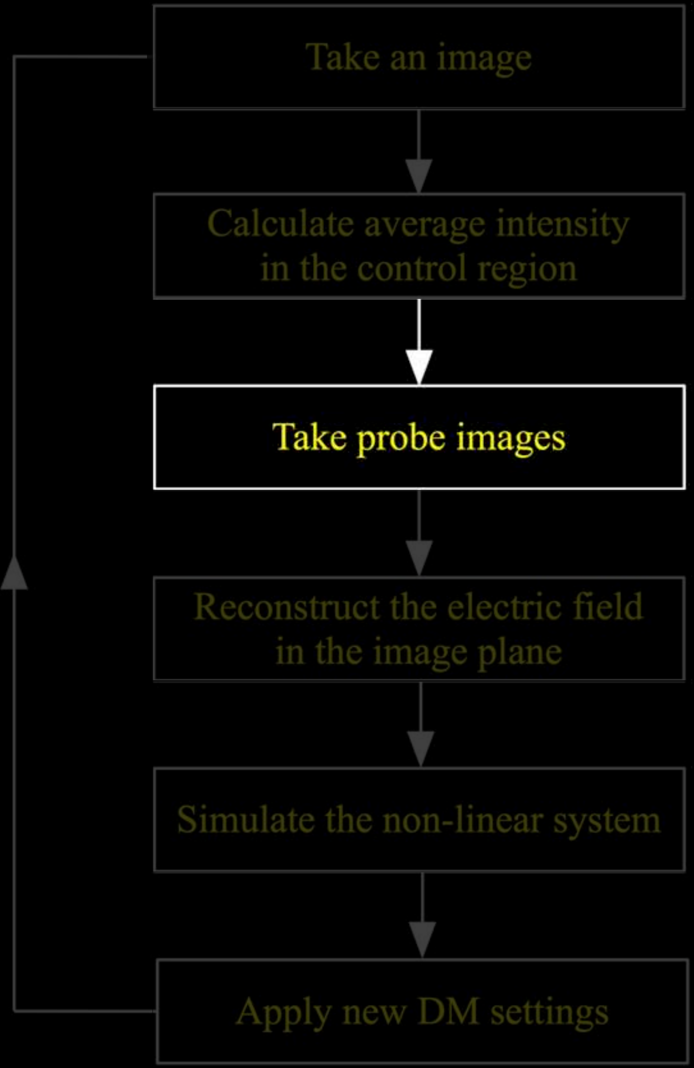
Measured image



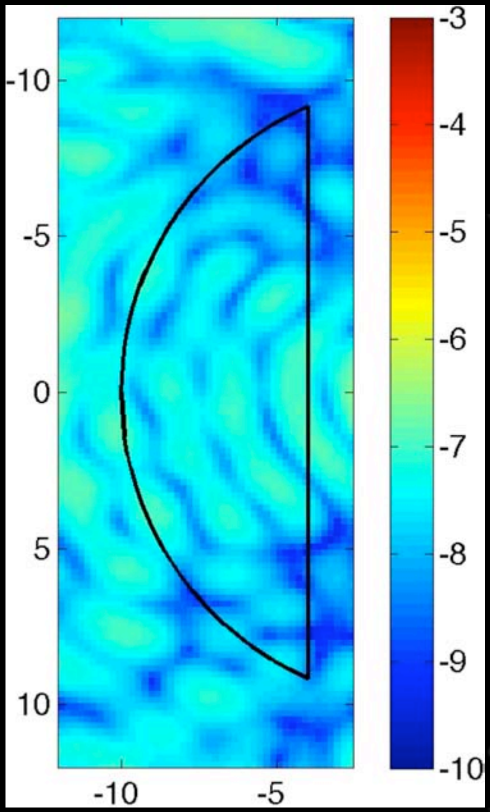
Measurements were taken at 2% around 800nm

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

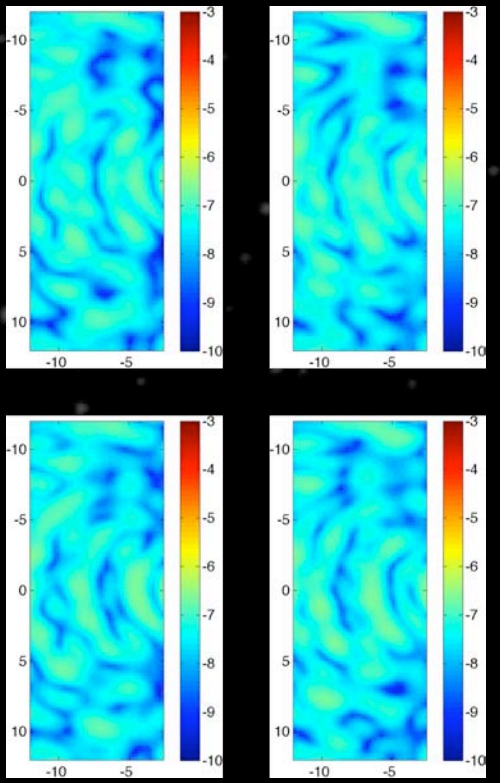
The algorithm



Measured image



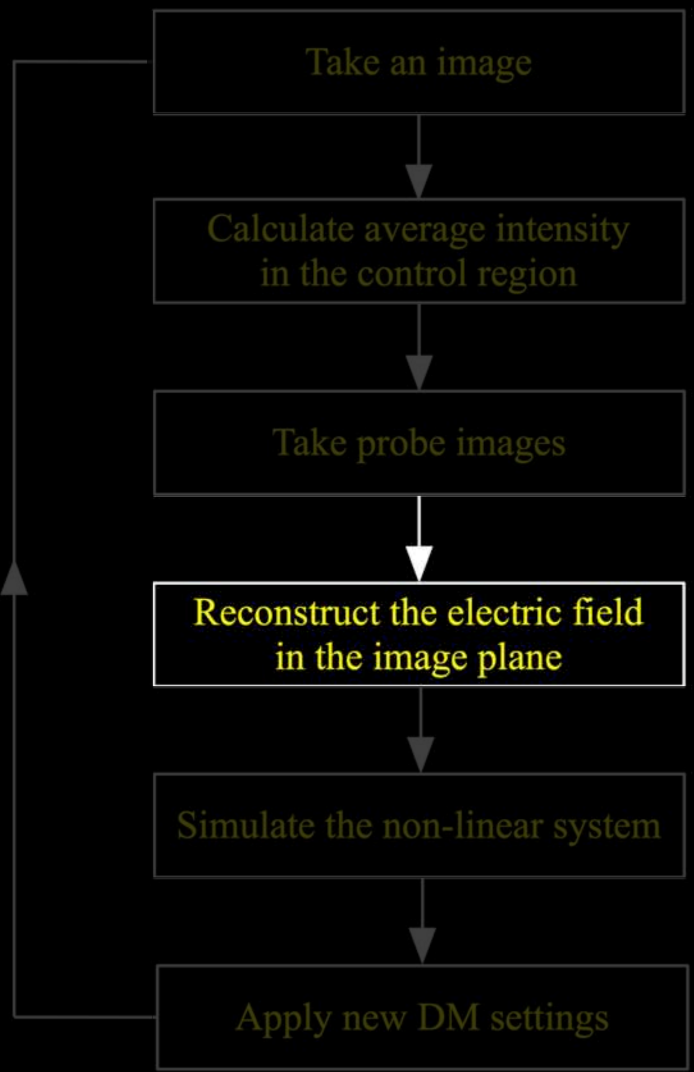
Probe images



Measurements were taken at 2% around 800nm

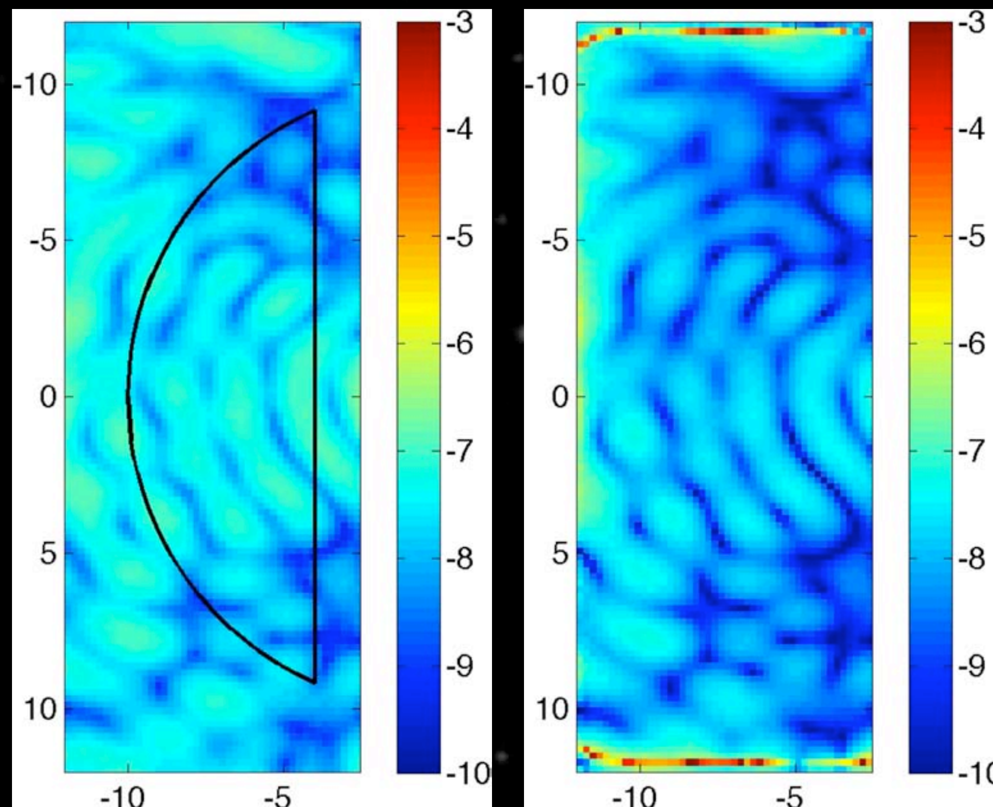
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The algorithm



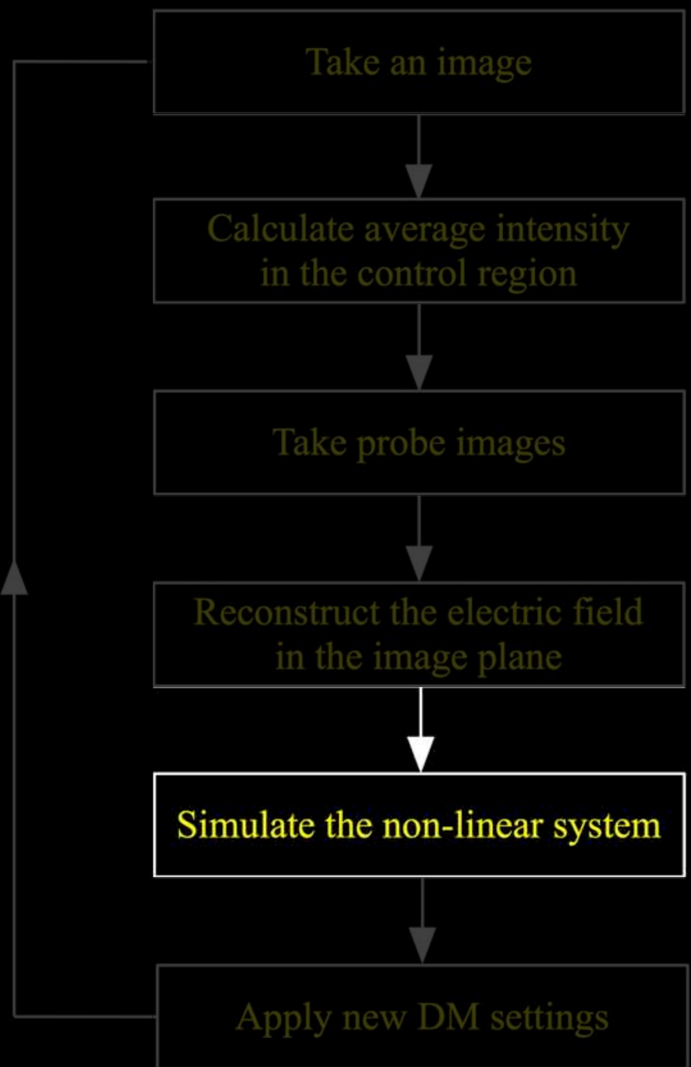
Measured image

Intensity from reconstructed electric field

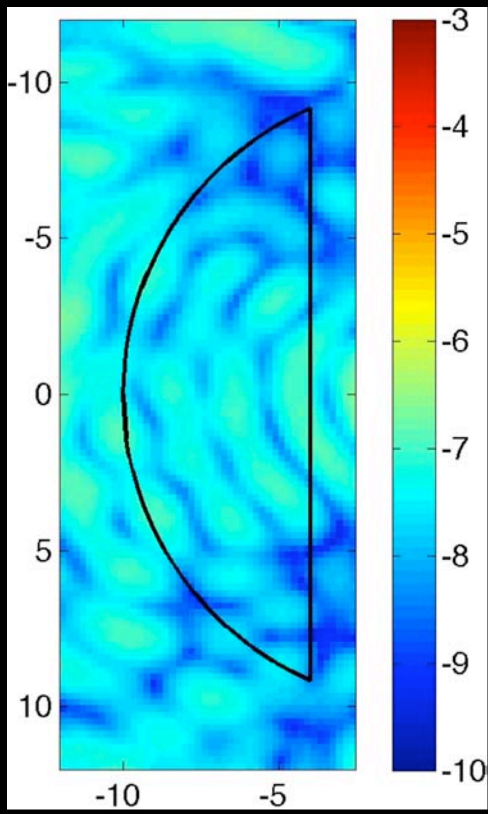


Measurements were taken at 2% around 800nm

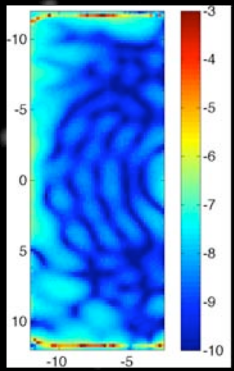
The algorithm



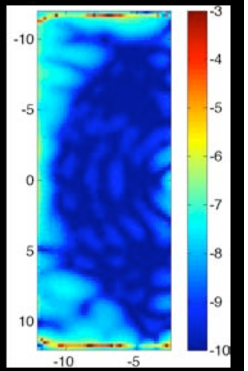
Measured image



After one iteration



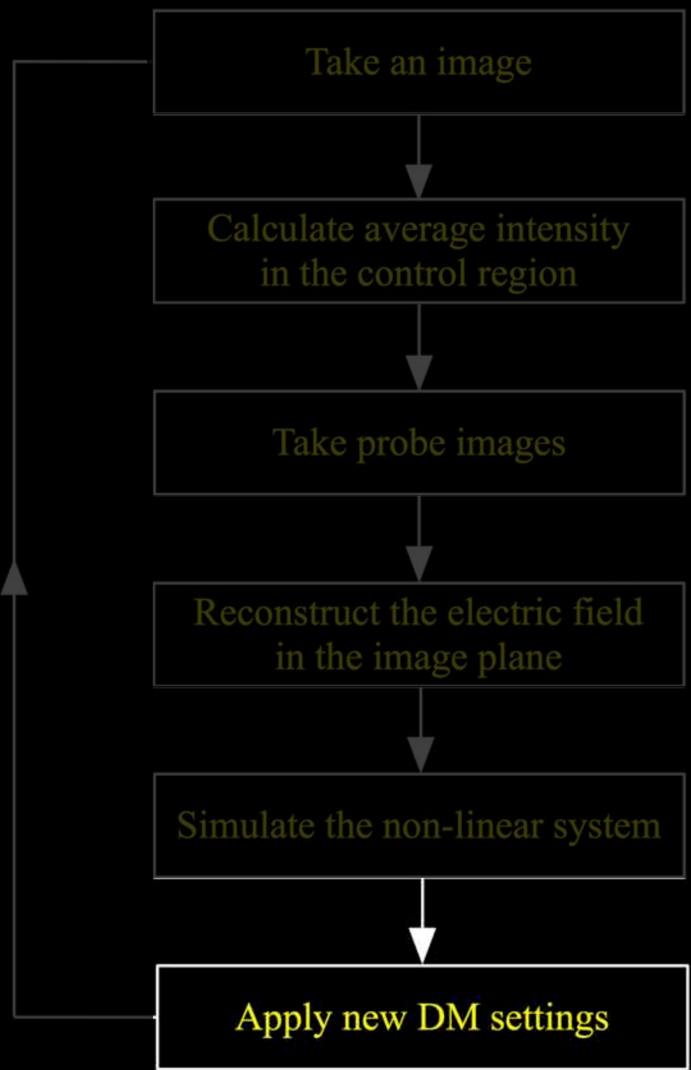
After five iterations



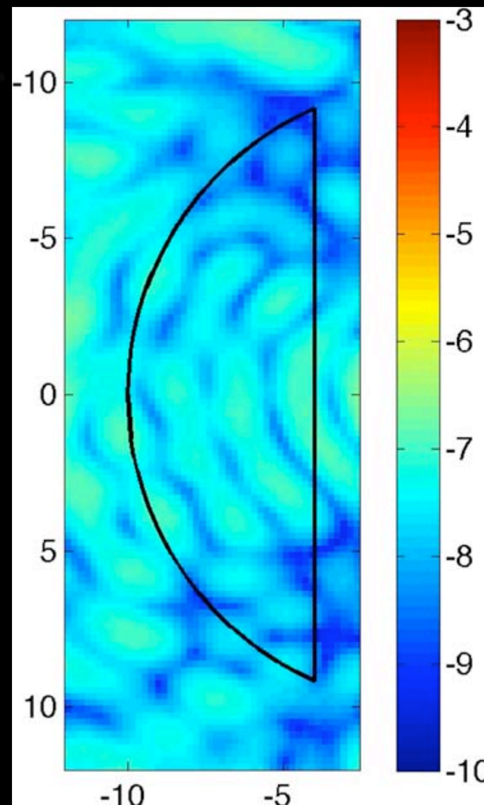
Measurements were taken at 2% around 800nm

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

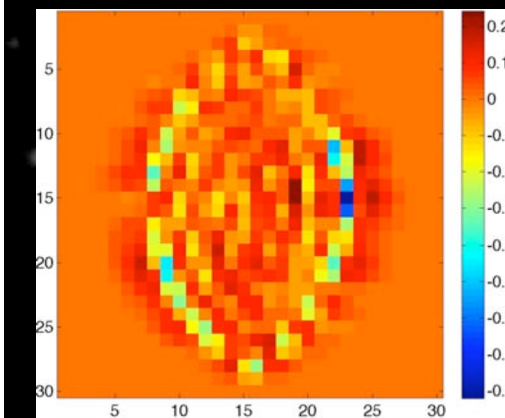
The algorithm



Measured image



Change in DM actuators



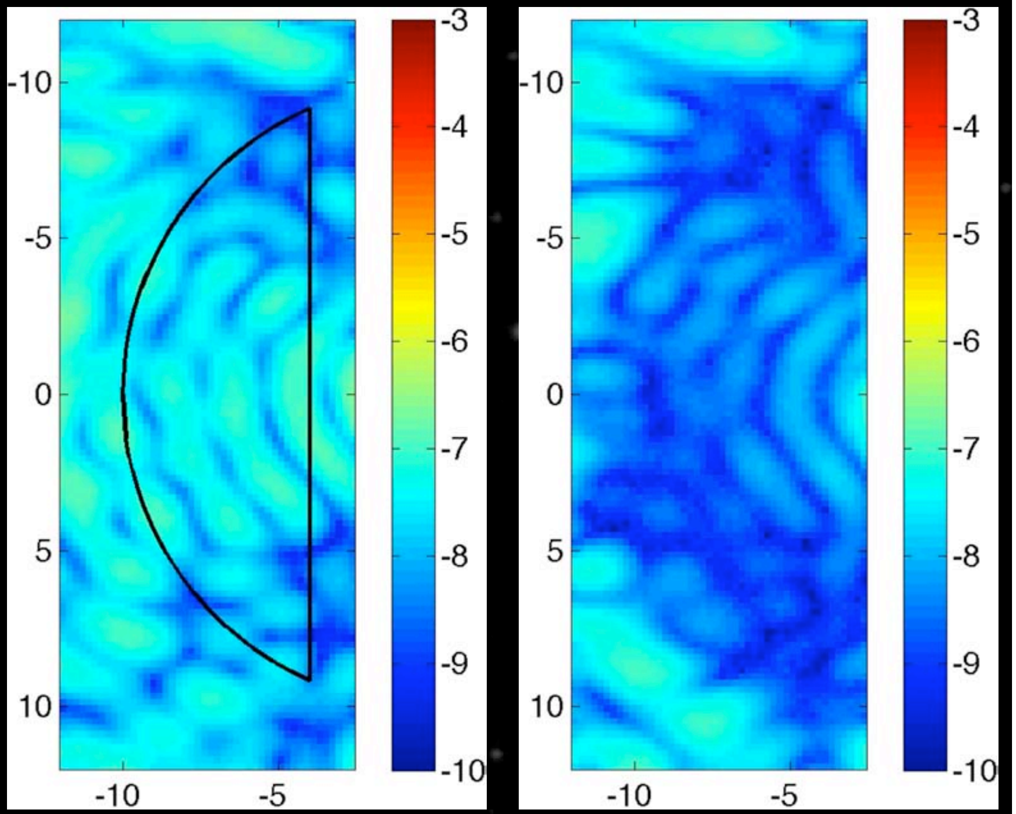
Measurements were taken at 2% around 800nm

The algorithm

1. Take an image
2. Calculate average intensity in the control region
3. Take probe images
4. Reconstruct the electric field in the image plane
5. Simulate the non-linear system
6. Apply new DM settings

(Loop back to "Take an image")

Measured image | New measured image



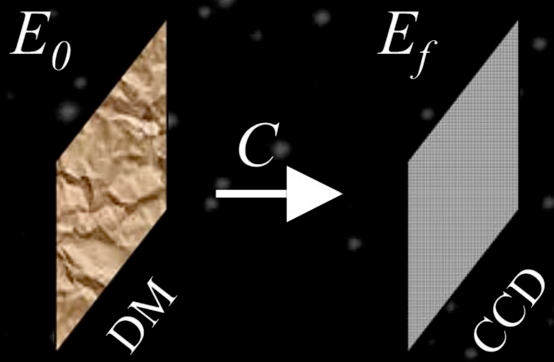
Measurements were taken at 2% around 800nm

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The reconstruction stage

Let C be the transformation between the electric field in DM plane to the electric field in the image plane

$$E_f = C\left\{E_0\right\}$$



The reconstruction stage

Let C be the transformation between the electric field in DM plane to the electric field in the image plane

$$E_f = C\left\{E_0\right\}$$

For a corrected system with phase and amplitude aberrations, the electric field in the DM plane can be modeled as

$$E_0 = Ae^{\alpha+i\beta}e^{i\psi}$$

The reconstruction stage

Let C be the transformation between the electric field in DM plane to the electric field in the image plane

$$E_f = C\left\{E_0\right\}$$

For a corrected system with phase and amplitude aberrations, the electric field in the DM plane can be modeled as

$$E_0 = Ae^{\alpha + i\beta}e^{i\psi}$$

The electric field in the image plane can be approximated as

$$E_f \approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\}$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The reconstruction stage

Let C be the transformation between the electric field in DM plane to the electric field in the image plane

$$E_f = C\left\{E_0\right\}$$

For a corrected system with phase and amplitude aberrations, the electric field in the DM plane can be modeled as

$$E_0 = Ae^{\alpha+i\beta}e^{i\psi}$$

The electric field in the image plane can be approximated as

$$E_f \approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\}$$

Then, the intensity in the image plane is given by

$$I_k \approx \left|C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi_k\right\}\right|^2$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The reconstruction stage

Suppose we deform the DM in pairs of conjugate shapes, then, the pair of probe intensity measurements is given by

$$I_k^+ \approx \left| C\left\{ A e^{\alpha + i\beta} \right\} + iC\left\{ A\psi_k \right\} \right|^2$$
$$I_k^- \approx \left| C\left\{ A e^{\alpha + i\beta} \right\} - iC\left\{ A\psi_k \right\} \right|^2$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The reconstruction stage

Suppose we deform the DM in pairs of conjugate shapes, then, the pair of probe intensity measurements is given by

$$I_k^+ \approx \left|C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi_k\right\}\right|^2$$
$$I_k^- \approx \left|C\left\{Ae^{\alpha+i\beta}\right\} - iC\left\{A\psi_k\right\}\right|^2$$

For each pair, at any given point in the image plane,

$$I_k^+ - I_k^- = 2C^*\left\{Ae^{\alpha+i\beta}\right\}C\left\{A\psi_k\right\} + 2C\left\{Ae^{\alpha+i\beta}\right\}C^*\left\{A\psi_k\right\}$$

The reconstruction stage

Suppose we deform the DM in pairs of conjugate shapes, then, the pair of probe intensity measurements is given by

$$I_k^+ \approx \left| C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi_k\right\}\right|^2$$
$$I_k^- \approx \left| C\left\{Ae^{\alpha+i\beta}\right\} - iC\left\{A\psi_k\right\}\right|^2$$

For each pair, at any given point in the image plane,

$$I_k^+ - I_k^- = 2C^*\left\{Ae^{\alpha+i\beta}\right\}C\left\{A\psi_k\right\} + 2C\left\{Ae^{\alpha+i\beta}\right\}C^*\left\{A\psi_k\right\}$$

Using all measurements, we can estimate the real and imaginary parts of the electric field in the image plane (at any given point) using:

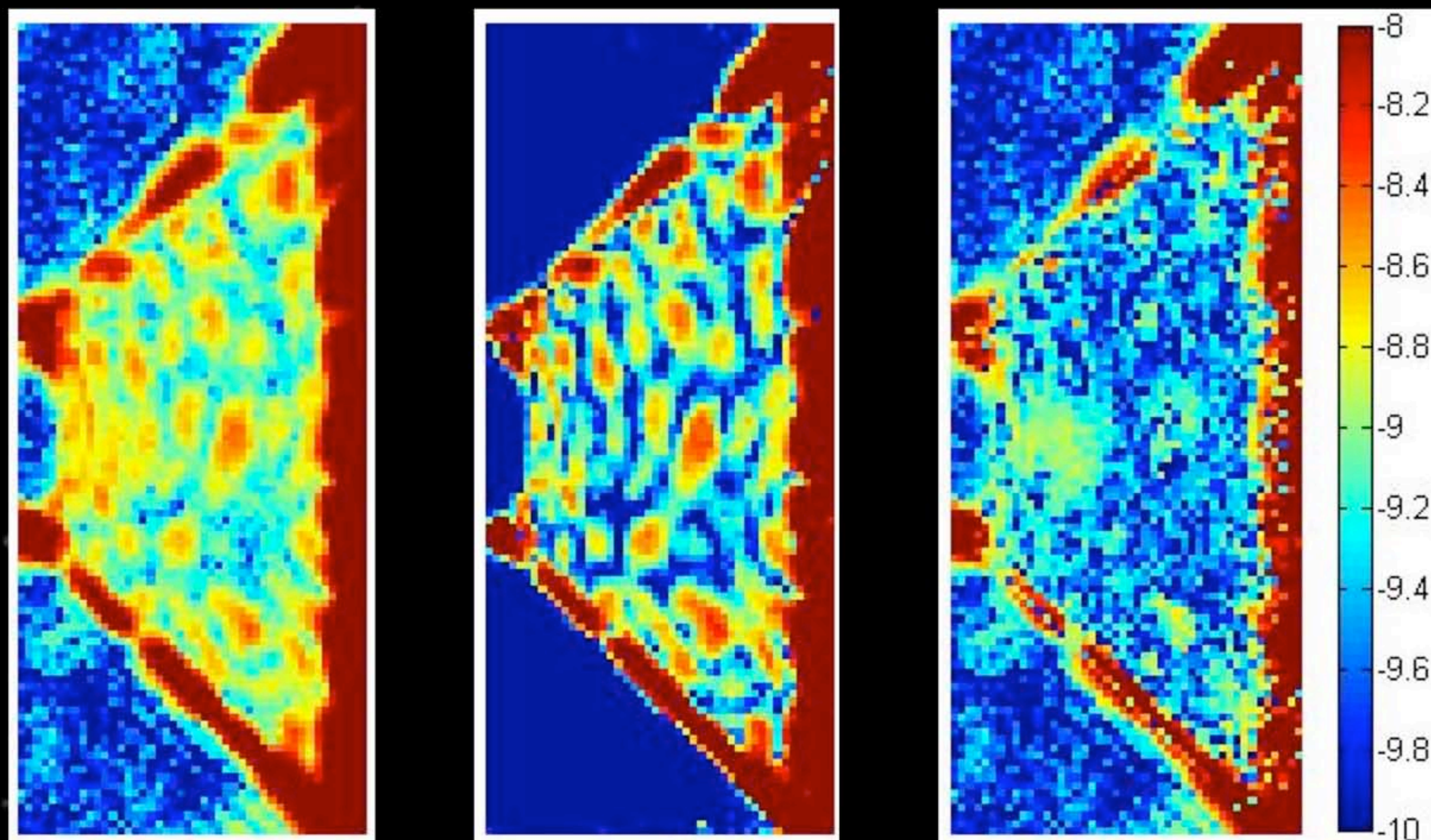
This method is more robust than the previous one we used on the Princeton testbed

$$\begin{bmatrix} I_1^+ - I_1^- \\ \vdots \\ I_k^+ - I_k^- \end{bmatrix} = \frac{1}{4}\begin{bmatrix} \Re\{C\{A\psi_1\}\} & \Im\{C\{A\psi_1\}\} \\ \vdots & \vdots \\ \Re\{C\{A\psi_k\}\} & \Im\{C\{A\psi_k\}\} \end{bmatrix}\begin{bmatrix} \Re\left\{C\left\{Ae^{\alpha+i\beta}\right\}\right\} \\ \Im\left\{C\left\{Ae^{\alpha+i\beta}\right\}\right\} \end{bmatrix}$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

The reconstruction stage

Intensity measurements (planet added analytically)	Computed intensity from reconstructed EF	Difference



The added planet simulates a planet, 3 times bigger (in radius) than Earth in the habitable zone of a G-type star 7-8 parsecs away for a 4m telescope imaging at 750nm.

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Electric field conjugation

The electric field in the image plane

$$E_f \approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\}$$
$$= E_{ab} + iC\left\{A\psi\right\}$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Electric field conjugation

The electric field in the image plane

$$\begin{aligned} E_f &\approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\} \\ &= E_{ab} + iC\left\{A\psi\right\} \end{aligned}$$

Assuming linearity of the DM:

$$C\left\{A\psi\right\} = G\bar{a}$$

Electric field conjugation

The electric field in the image plane

$$\begin{aligned} E_f &\approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\} \\ &= E_{ab} + iC\left\{A\psi\right\} \end{aligned}$$

Assuming linearity of the DM:

$$C\left\{A\psi\right\} = G\bar{a}$$

Electric field conjugation:

$$\boxed{G\bar{a} = iE_{ab}}$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Electric field conjugation

The electric field in the image plane

$$E_f \approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\}$$
$$= E_{ab} + iC\left\{A\psi\right\}$$

Assuming linearity of the DM:

$$C\left\{A\psi\right\} = G\bar{a}$$

Electric field conjugation:

$$G\bar{a} = E$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Electric field conjugation

The electric field in the image plane

$$\begin{aligned} E_f &\approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\} \\ &= E_{ab} + iC\left\{A\psi\right\} \end{aligned}$$

Assuming linearity of the DM:

$$C\left\{A\psi\right\} = G\bar{a}$$

Electric field conjugation:

$$G\bar{a} = E$$

How do we construct the G matrix?

Each column of G is (analytically) poking an actuator and recording its effect in the image plane. Under the superposition assumption, the effect of the DM in the image plane will be $G\bar{a}$

Electric field conjugation

The electric field in the image plane

$$E_f \approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\}$$
$$= E_{ab} + iC\left\{A\psi\right\}$$

Assuming linearity of the DM:

$$C\left\{A\psi\right\} = G\bar{a}$$

Electric field conjugation:

$$\boxed{G\bar{a} = E}$$

Solution (assuming real valued):

$$\bar{a} = \begin{bmatrix} \Re\left\{G\right\} \\ \cdots \\ \Im\left\{G\right\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\left\{E\right\} \\ \cdots \\ \Im\left\{E\right\} \end{bmatrix}$$

Electric field conjugation

The electric field in the image plane

$$\begin{aligned} E_f &\approx C\left\{Ae^{\alpha+i\beta}\right\} + iC\left\{A\psi\right\} \\ &= E_{ab} + iC\left\{A\psi\right\} \end{aligned}$$

Assuming linearity of the DM:

$$C\left\{A\psi\right\} = G\bar{a}$$

Electric field conjugation:

$$G\bar{a} = E$$

Solution (assuming real valued):

$$\bar{a} = \begin{bmatrix} \Re\{G\} \\ \cdots \\ \Im\{G\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{E\} \\ \cdots \\ \Im\{E\} \end{bmatrix}$$

Interpretation:

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|G\bar{a} - E\|^2$$

Regularization

Regional weighting in the image plane:

$$\bar{a} = \begin{bmatrix} \Re\{WG\} \\ \cdots \\ \Im\{WG\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{WE\} \\ \cdots \\ \Im\{WE\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|WG\bar{a} - WE\|^2$$

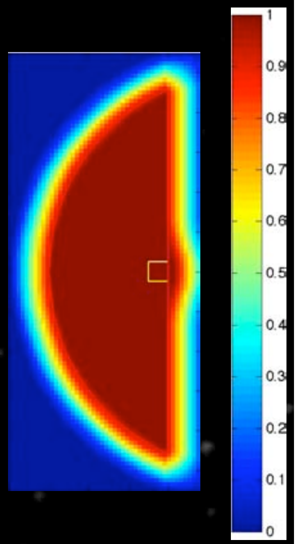
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Regularization

Regional weighting in the image plane:

$$\bar{a} = \begin{bmatrix} \Re\{WG\} \\ \cdots \\ \Im\{WG\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{WE\} \\ \cdots \\ \Im\{WE\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|WG\bar{a} - WE\|^2$$

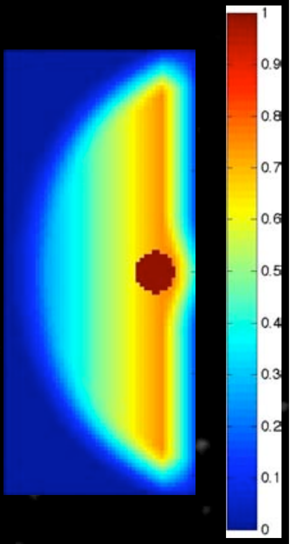


Regularization

Regional weighting in the image plane:

$$\bar{a} = \begin{bmatrix} \Re\{WG\} \\ \cdots \\ \Im\{WG\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{WE\} \\ \cdots \\ \Im\{WE\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|WG\bar{a} - WE\|^2$$



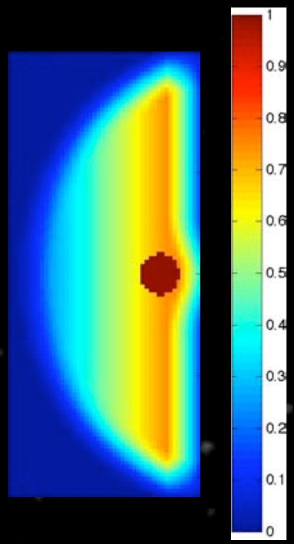
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Regularization

Regional weighting in the image plane:

$$\bar{a} = \begin{bmatrix} \Re\{WG\} \\ \cdots \\ \Im\{WG\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{WE\} \\ \cdots \\ \Im\{WE\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|WG\bar{a} - WE\|^2$$



Actuators stroke:

$$\bar{a} = \begin{bmatrix} \Re\{G\} \\ \cdots \\ \Im\{G\} \\ \cdots \\ \mu \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{E\} \\ \cdots \\ \Im\{E\} \\ \cdots \\ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{bmatrix}$$

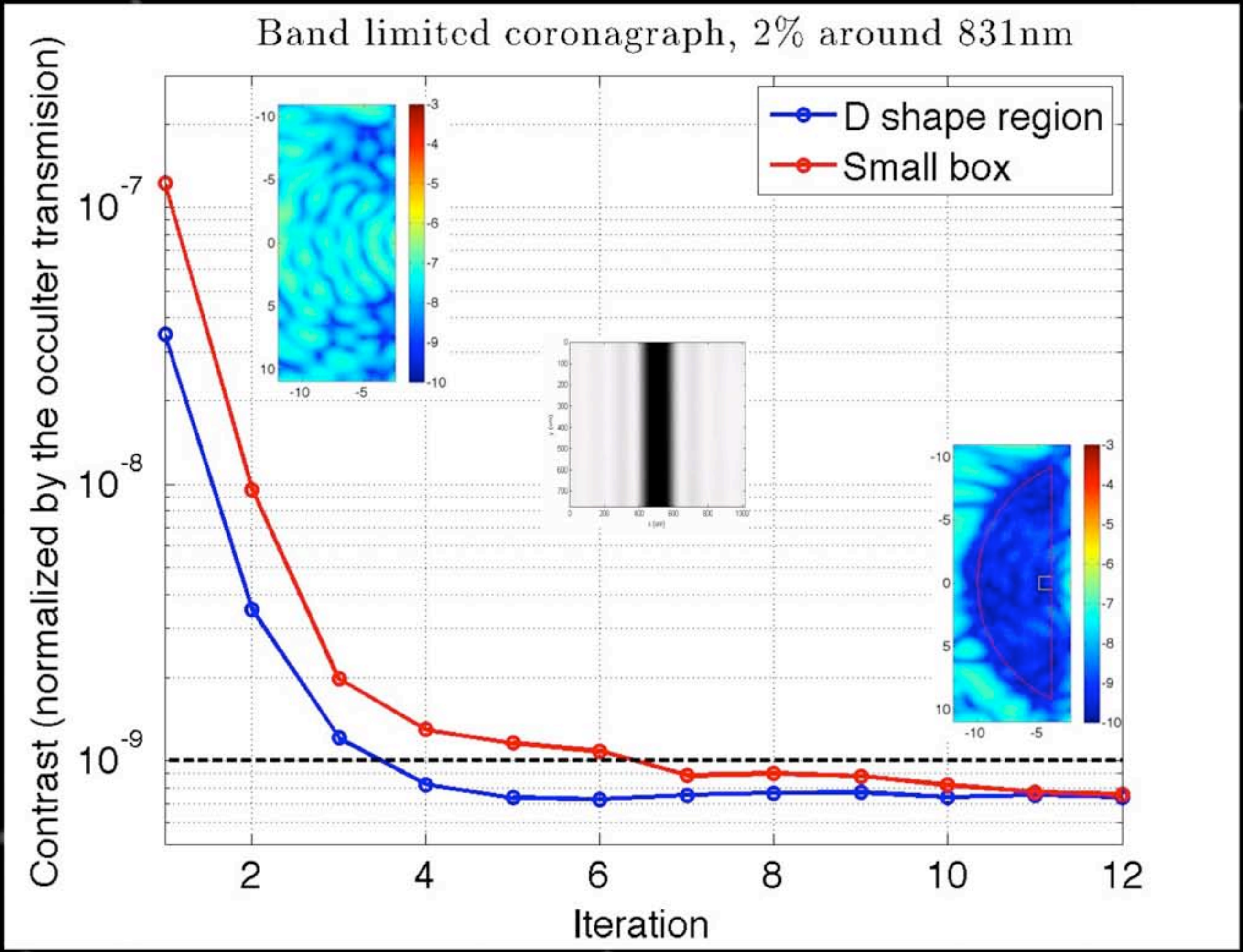
EFC

Actuators regularization

μ determines the weighting between contrast and actuators stroke

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|G\bar{a} - E\|^2 + \mu^2 \|\bar{a}\|^2$$

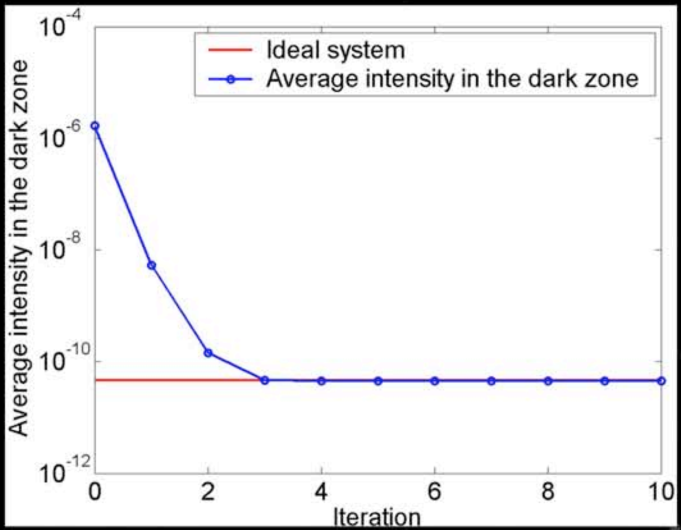
Experimental results



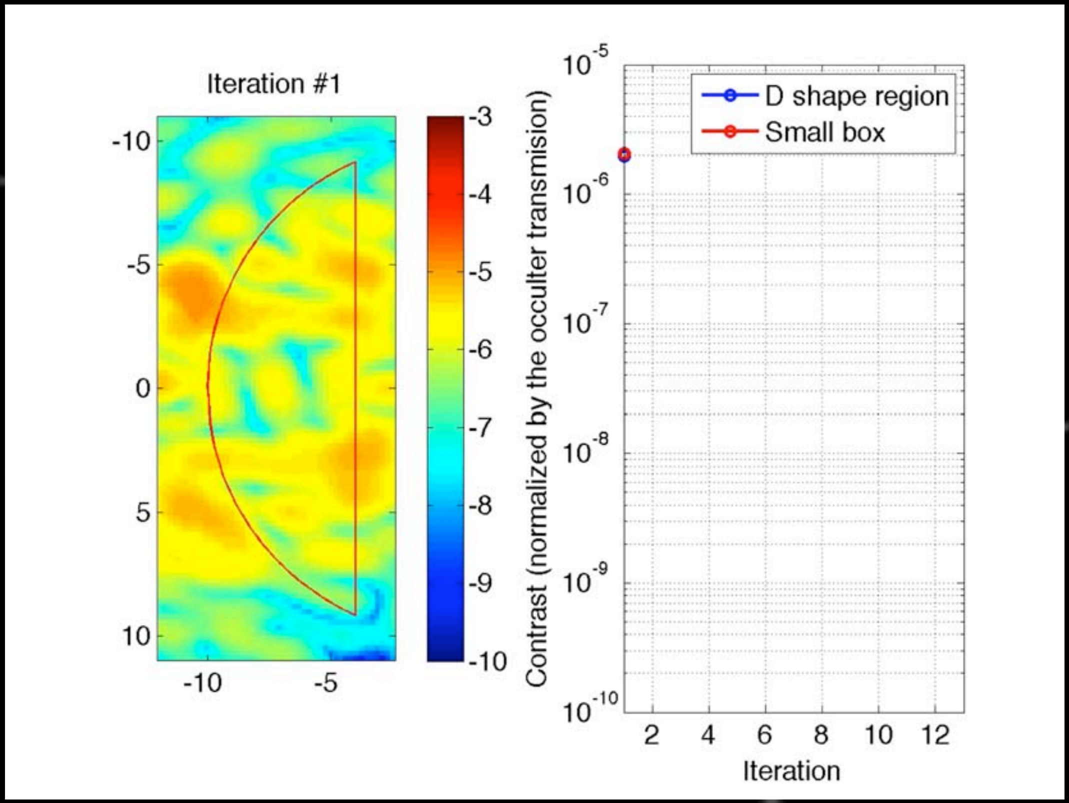
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Experimental results

Simulations shown last year



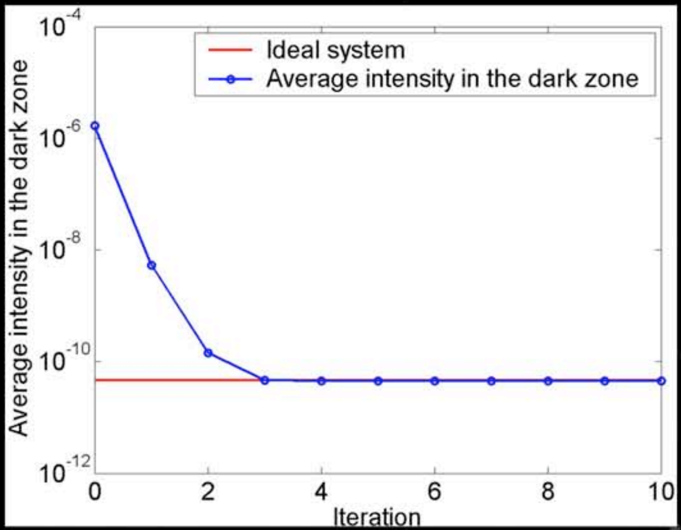
Experimental results



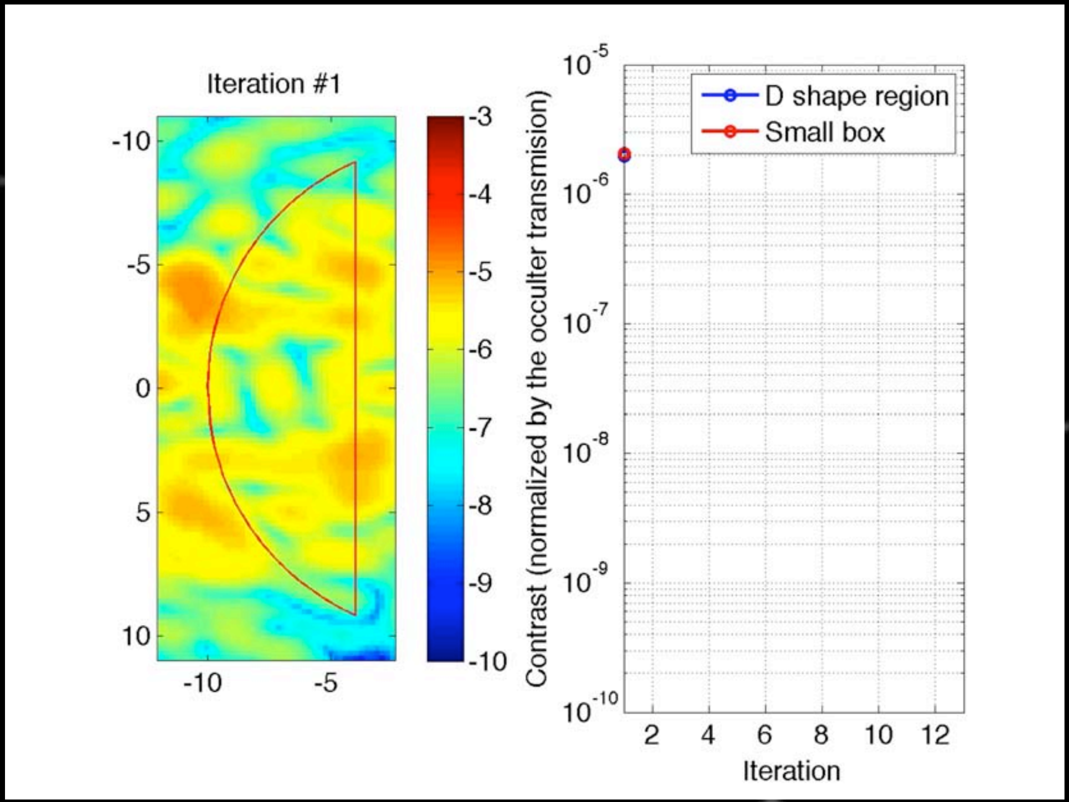
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Experimental results

Simulations shown last year

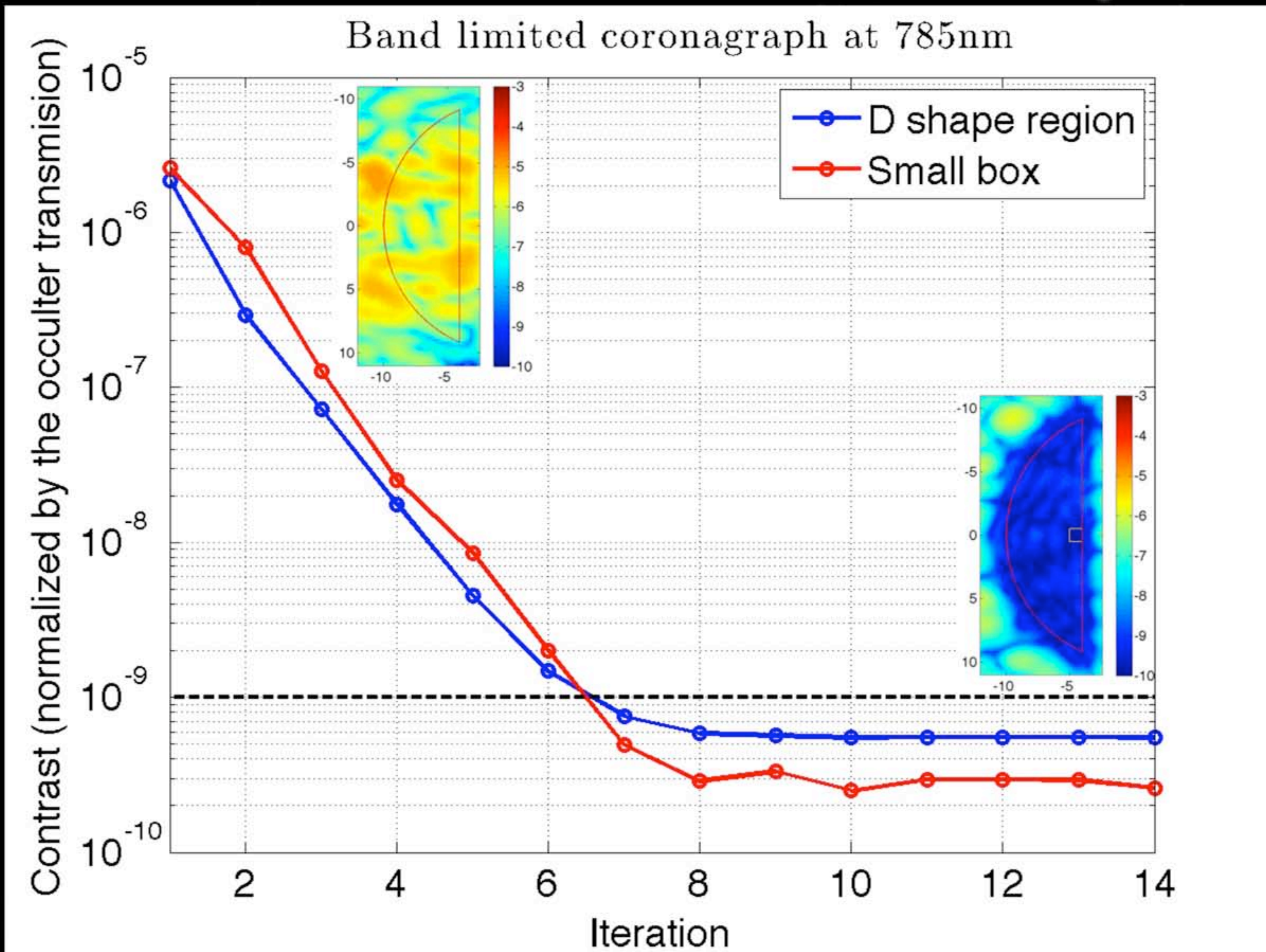


Experimental results



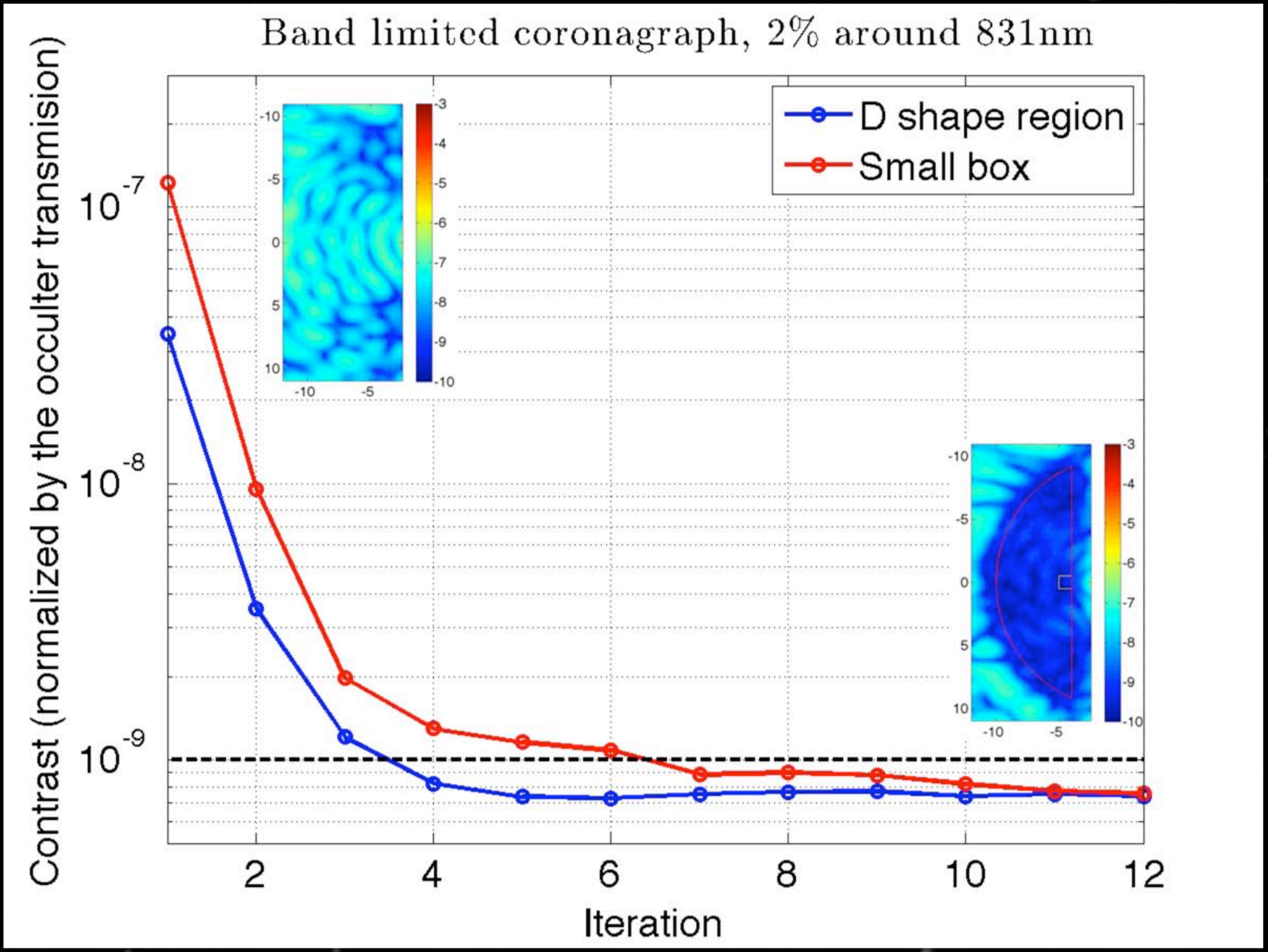
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Experimental results

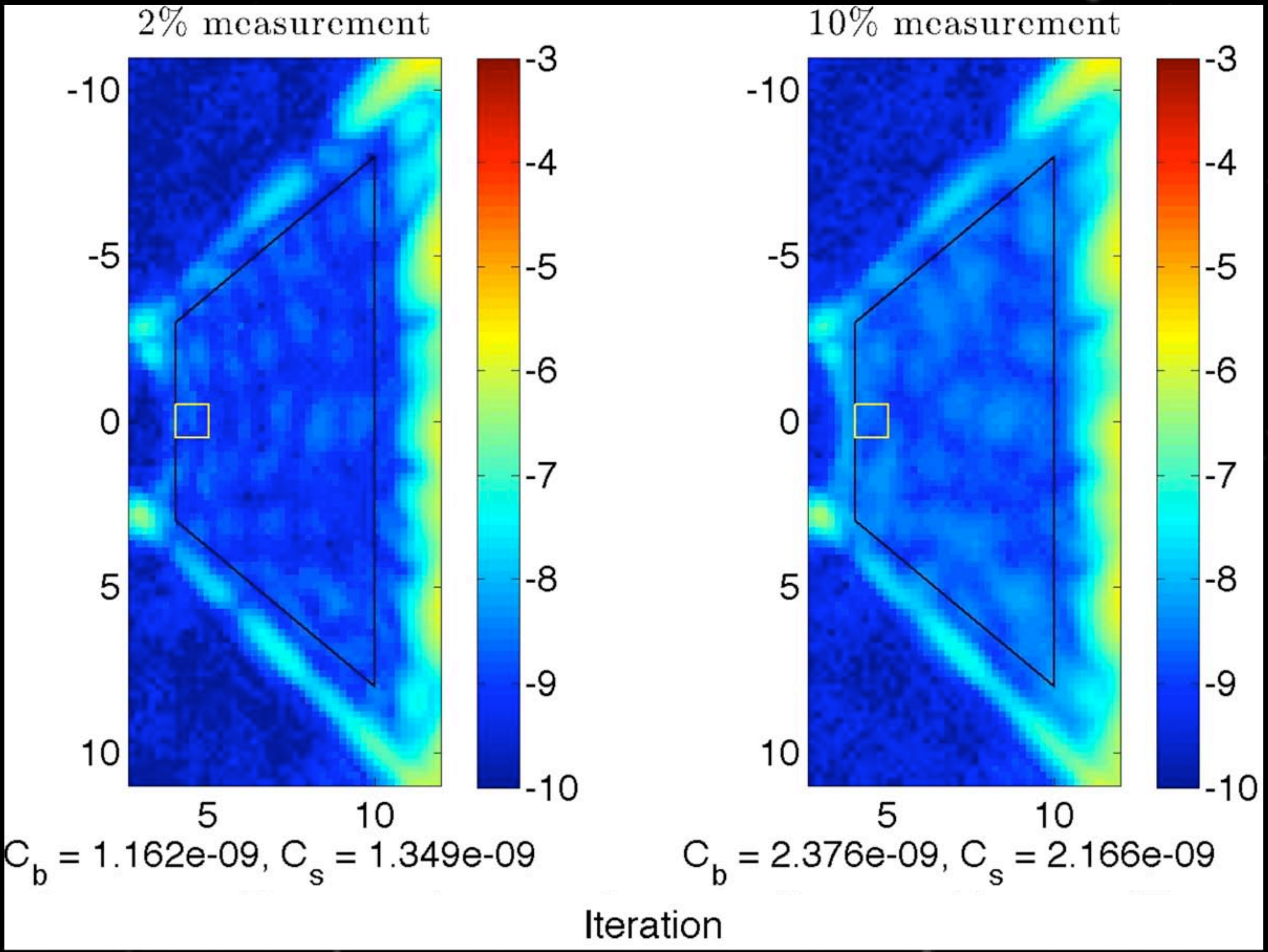


National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Experimental results



Experimental results



Broadband correction

Reconstruct the electric field at different wavelengths… and then,

$$\bar{a} = \begin{bmatrix} \Re\{W_{\lambda_1} G_{\lambda_1}\} \\ \Im\{W_{\lambda_1} G_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k} G_{\lambda_k}\} \\ \Im\{W_{\lambda_k} G_{\lambda_k}\} \\ \cdots \\ \mu \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{W_{\lambda_1} E_{\lambda_1}\} \\ \Im\{W_{\lambda_1} E_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k} E_{\lambda_k}\} \\ \Im\{W_{\lambda_k} E_{\lambda_k}\} \\ \cdots \\ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{bmatrix}$$

Electric field conjugation

Actuators regularization

$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|G_{\lambda_1}\bar{a} - E_{\lambda_1}\|^2 + \|G_{\lambda_2}\bar{a} - E_{\lambda_2}\|^2 + \cdots + \|G_{\lambda_k}\bar{a} - E_{\lambda_k}\|^2 + \mu^2 \|\bar{a}\|^2$$

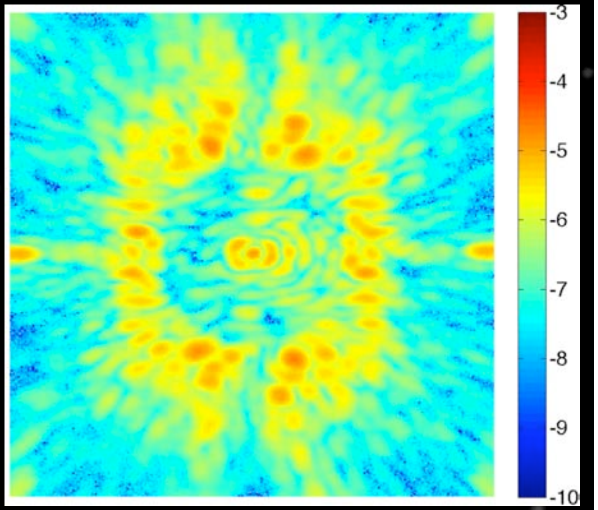
Broadband correction

Reconstruct the electric field at different wavelengths… and then,

$$\bar{a} = \begin{bmatrix} \Re\{W_{\lambda_1} G_{\lambda_1}\} \\ \Im\{W_{\lambda_1} G_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k} G_{\lambda_k}\} \\ \Im\{W_{\lambda_k} G_{\lambda_k}\} \\ \cdots \\ \mu \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{W_{\lambda_1} E_{\lambda_1}\} \\ \Im\{W_{\lambda_1} E_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k} E_{\lambda_k}\} \\ \Im\{W_{\lambda_k} E_{\lambda_k}\} \\ \cdots \\ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{bmatrix}$$

Electric field conjugation

Actuators regularization



$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G_{\lambda_1}\bar{a} - E_{\lambda_1}\|^2 + \|G_{\lambda_2}\bar{a} - E_{\lambda_2}\|^2 + \cdots + \|G_{\lambda_k}\bar{a} - E_{\lambda_k}\|^2 + \mu^2 \|\bar{a}\|^2$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

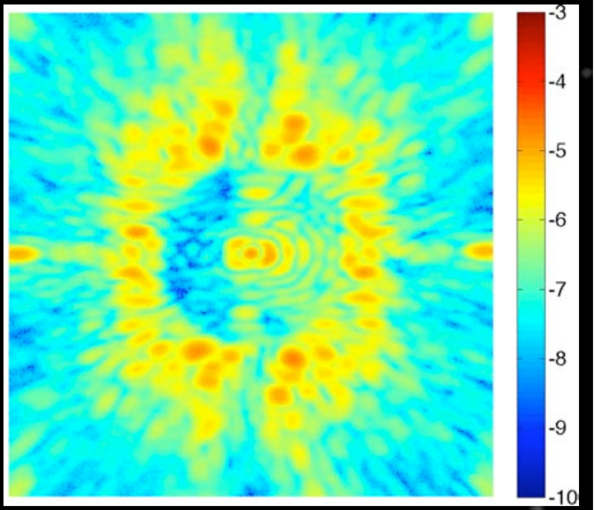
Broadband correction

Reconstruct the electric field at different wavelengths… and then,

$$\bar{a} = \begin{bmatrix} \Re\{W_{\lambda_1}G_{\lambda_1}\} \\ \Im\{W_{\lambda_1}G_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k}G_{\lambda_k}\} \\ \Im\{W_{\lambda_k}G_{\lambda_k}\} \\ \cdots \\ \mu \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{W_{\lambda_1}E_{\lambda_1}\} \\ \Im\{W_{\lambda_1}E_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k}E_{\lambda_k}\} \\ \Im\{W_{\lambda_k}E_{\lambda_k}\} \\ \cdots \\ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{bmatrix}$$

Electric field conjugation

Actuators regularization



$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|G_{\lambda_1}\bar{a} - E_{\lambda_1}\|^2 + \|G_{\lambda_2}\bar{a} - E_{\lambda_2}\|^2 + \cdots + \|G_{\lambda_k}\bar{a} - E_{\lambda_k}\|^2 + \mu^2\|\bar{a}\|^2$$

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

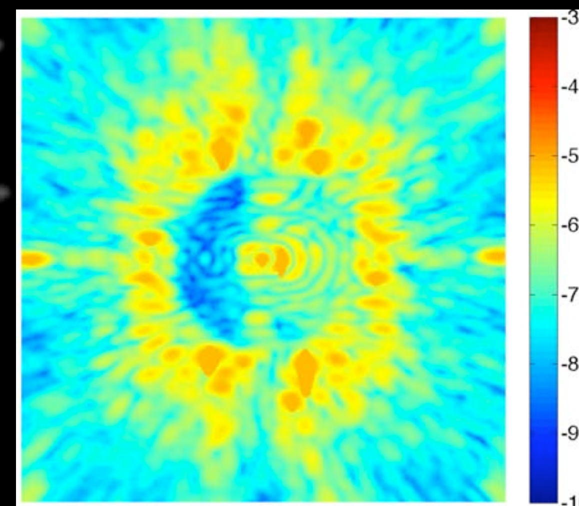
Broadband correction

Reconstruct the electric field at different wavelengths… and then,

$$\bar{a} = \begin{bmatrix} \Re\{W_{\lambda_1}G_{\lambda_1}\} \\ \Im\{W_{\lambda_1}G_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k}G_{\lambda_k}\} \\ \Im\{W_{\lambda_k}G_{\lambda_k}\} \\ \cdots \\ \mu \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{W_{\lambda_1}E_{\lambda_1}\} \\ \Im\{W_{\lambda_1}E_{\lambda_1}\} \\ \vdots \\ \vdots \\ \Re\{W_{\lambda_k}E_{\lambda_k}\} \\ \Im\{W_{\lambda_k}E_{\lambda_k}\} \\ \cdots \\ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{bmatrix}$$

Electric field conjugation

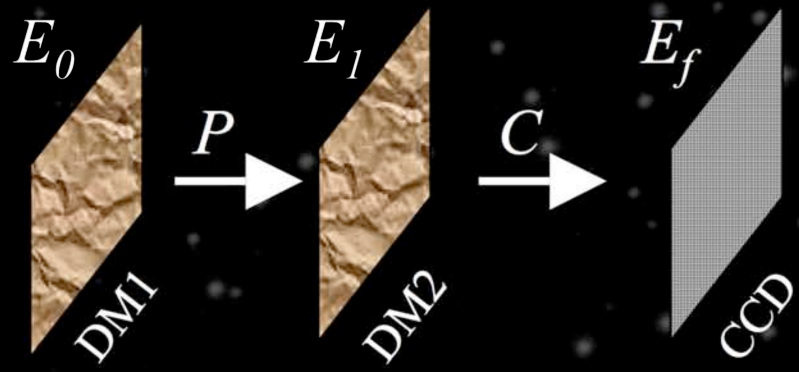
Actuators regularization



$$\bar{a}^* = \arg\min_{\bar{a}\in X} \|G_{\lambda_1}\bar{a} - E_{\lambda_1}\|^2 + \|G_{\lambda_2}\bar{a} - E_{\lambda_2}\|^2 + \cdots + \|G_{\lambda_k}\bar{a} - E_{\lambda_k}\|^2 + \mu^2 \|\bar{a}\|^2$$

Broadband correction

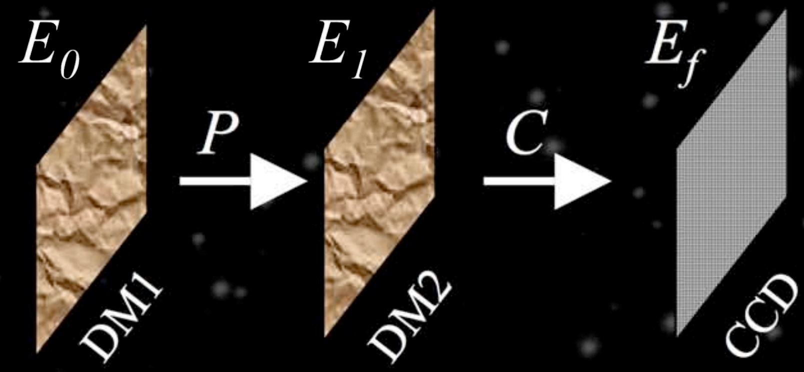
What about using more than one DM?



National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology

Broadband correction

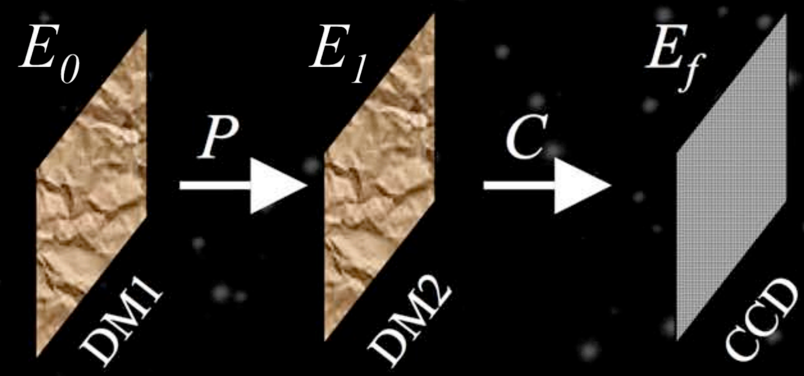
What about using more than one DM?



The formulation shown in the previous slide can be used for a multi-DM system. Its all about the *G* matrix!

Broadband correction

What about using more than one DM?



The formulation shown in the previous slide can be used for a multi-DM system. Its all about the *G* matrix!

Just like before, each column of the *G* matrix will be the effect in the image plane of a single DM actuator. Actuators from DM1 will have the additional effect from propagating between E_0 and E_1.

Summary

- 2.37×10^{-9} contrast for SPC at 10% wavelength band around 800nm.
- 7×10^{-10} contrast for BLC at 2% wavelength band around 832nm.
- 7×10^{-10} contrast for BLC at monochromatic light around 785nm.