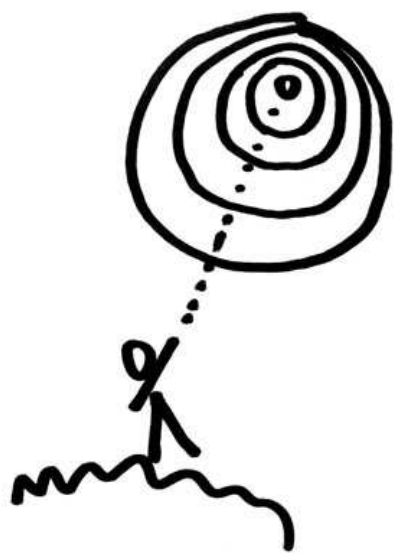
Astro/EPS C12 (2008) – Mike Wong
Problem Set 3 Solution Guide

1 Comet baseball

Problem 1A
Super easy with the density formula, and the volume of a sphere:

$$V = \frac{4\ \pi\ r^3}{3} = \frac{4\ \pi\ (60\ \text{km} = 60000\ \text{m})^3}{3} = 9.0 \times 10^{14}\ \text{m}^3$$
$$m = \rho\ V = 1000\ \text{kg m}^{-3} \times 9.0 \times 10^{14}\ \text{m}^3 = 9.0 \times 10^{17}\ \text{kg}$$

Problem 1B
Radio waves moving outward from the baseball are blueshifted forward in the direction of the ball's travel, and redshifted behind. So I would detect a redshifted signal.



Problem 1C
If the 30 m sec^{-1} velocity of the ball is greater than the escape velocity, then the ball can escape. So calculate the escape velocity:

$$v_{\text{esc.}} = \sqrt{\frac{2\ G\ m}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\,\text{sec}^{-2}) \times (9.0 \times 10^{17}\ \text{kg})}{60000\ \text{m}}} = 45\ \text{m}\,\text{sec}^{-1}$$

The ball WILL return.

2 Pressure and vapor pressure

Problem 2A

$$\mu_{\text{N}_2} = 2\ \mu_{\text{N}} = 2 \times 14 = 28$$
$$\mu_{\text{H}_2\text{O}} = 2\ \mu_{\text{H}} + \mu_{\text{O}} = 2 \times 1 + 16 = 18$$

$$\mu_{\text{air}} = 0.78\ \mu_{\text{N}_2} + 0.21\ \mu_{\text{O}_2} + 0.01\ \mu_{\text{H}_2\text{O}} = 21.8 + 6.7 + 0.2 = 28.7$$

Problem 2B
First calculate the scale height at 8 km, 11 km, and 40 km. For convenience, we use the composition of moist air (part 2A) for the molecular weight of air each time, even though there will be less water vapor at high altitudes. We also neglect the variation of gravitational acceleration with altitude, which is a small effect so close to the Earth's surface. Then the only thing that changes in the scale height formula is the temperature used.

$$H_{(8\ \text{km})} = \frac{k\ T}{\mu_{\text{air}}\ m_u\ g} = \frac{(1.38 \times 10^{-23}\ \text{J K}^{-1} = 1.38 \times 10^{-23}\ \text{kg m}^2\,\text{sec}^{-2}\ \text{K}^{-1})\ 250\ \text{K}}{28.7\ (1.66 \times 10^{-27}\ \text{kg})\ 9.8\ \text{m}\,\text{sec}^{-2}}$$

$$H_{(8\ \text{km})} = 29.6\ \text{m K}^{-1} \times 250\ \text{K} = 7400\ \text{m} = 7.4\ \text{km}$$

$$H_{(11\ \text{km})} = 29.6\ \text{m K}^{-1} \times 250\ \text{K} = 7.4\ \text{km}$$
$$H_{(40\ \text{km})} = 29.6\ \text{m K}^{-1} \times 240\ \text{K} = 7100\ \text{m} = 7.1\ \text{km}$$

With these scale heights, we can calculate the pressure at each altitude. I use a ground-level pressure of 1000 mbar.

$$p_{(8\ \text{km})} = p_0\ \exp(-8\ \text{km}\ /\ H_{(8\ \text{km})}) = p_0\ \exp(-8\ \text{km}\ /\ 7.4\ \text{km}) = 1000\ \text{mbar}\ e^{-1.08} = 340\ \text{mbar}$$
$$p_{(11\ \text{km})} = p_0\ \exp(-11\ \text{km}\ /\ H_{(11\ \text{km})}) = 1000\ \text{mbar}\ \exp(-11\ \text{km}\ /\ 7.4\ \text{km}) = 230\ \text{mbar}$$

$$p_{(40\ \mathrm{km})} = p_0\ \exp(-40\ \mathrm{km}\ /\ H_{(40\ \mathrm{km})}) = 1000\ \mathrm{mbar}\ \exp(-40\ \mathrm{km}\ /\ 7.1\ \mathrm{km}) = 3.6\ \mathrm{mbar}$$

Problem 2C

In the Himalayas, the pressure is around 340 mbar, or 0.34 bar. The saturation vapor curve shows that the saturation vapor pressure is equal to 0.34 bar when the temperature is 345 K, so this is the boiling temperature of water in the Himalayas. A temperature of 345 K is equivalent to 72° C, so the boiling temperature of water is 18° C lower than it is at sea level.

At passenger aircraft cruising altitude, the pressure is about 230 mbar. The saturation vapor curve shows that the saturation vapor pressure is equal to 0.23 bar when the temperature is about 337 K, so this is the boiling temperature of water at cruising altitude. This is equivalent to 64° C.

Problem 2D

In part 2C, we saw that as altitude (and pressure) decreased, the boiling temperature went down. Eventually, if pressure is low enough, the boiling temperature would be as low as 1° C. Cold water would boil at low pressures! At pressures lower than about 6 mbar (from the diagram), only solid water (ice) could exist.

Another way of thinking about it: At 40 km, I found a pressure of 3.6 mbar, so the partial pressure of water at this level must be less than or equal to 3.6 mbar, depending on how much of the air is made up of water. I can draw a line across the phase diagram at 0.0036 bar (3.6 mbar), and the partial pressure of water will have to be below that line. Then, for temperatures higher than about 268 K, only gas can exist. For temperatures lower than 268 K, you can have ice and gas, but no temperature allows for liquid water to exist at those pressures any more.