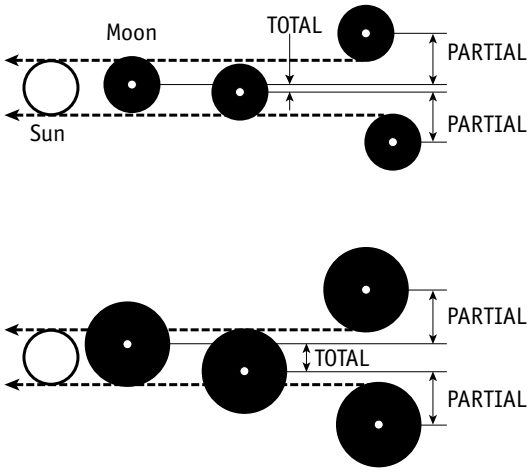
Astro/EPS C12 (2008) – Mike Wong
Problem Set 4 Solution Guide

1 Eclipse

Problem 1A
Solar eclipses only happen when the Moon is **in** the Earth's orbital plane, so that the Sun, Moon, and Earth are lined up and the Moon's shadow falls on the Earth. But because there is a 5° tilt between the Moon's orbital plane and the Earth's orbital plane, the Moon is not usually in the Earth's orbital plane. It crosses the Earth's orbital plane twice during its monthly orbit around the Earth, but a solar eclipse occurs only when that crossing falls exactly during the new moon.

Problem 1B
A total solar eclipse occurs when the Sun is completely covered by the Moon. If the Moon were twice its current angular size, events which are now total solar eclipses would remain total solar eclipses, and some near-miss events which are now partial solar eclipses would become total solar eclipses. As for partial solar eclipses, a few would be reclassified as total solar eclipses, but even more partial solar eclipses would occur that would not have due to the moon's larger angular size (and hence the probability of its penumbra falling on the Earth).

The diagram below shows what the sky looks like from Earth. The new Moon is shown drifting across the Sun. The range of paths falling within the "total" zone increases by a lot (it more than doubles). The range of paths falling within the "partial" zone increases by only a little bit (about 5%).



2 Equilibrium temperatures

Problem 2A
Solar flux at Saturn's distance of 9.54 AU:

$$F_{\text{Sat}} = F_0 \ (\frac{r_0^2}{a^2}) = 1370 \text{ W m}^{-2} \ (\frac{1^2 \text{ AU}^2}{9.54^2 \text{ AU}^2}) = 15.1 \text{ W m}^{-2}$$

Problem 2B
The darker leading hemisphere will absorb a lot more sunlight. For a 1 m^2 patch of ground,

$$F_{\text{leading}} = (1 \ - \ A_{\text{leading}})F_{\text{Sat}} = (1 - 0.03) \ 15.1 \text{ W m}^{-2} = 14.6 \text{ W m}^{-2}$$
$$F_{\text{trailing}} = (1 \ - \ A_{\text{trailing}})F_{\text{Sat}} = (1 - 0.6) \ 15.1 \text{ W m}^{-2} = 6.0 \text{ W m}^{-2}$$

Problem 2C
First balance the incoming and outgoing energy per unit area, and solve for the temperature:

$$F = \sigma T^4$$
$$T^4 = F/\sigma$$
$$T = \sqrt[4]{F/\sigma}$$

If your calculator can't do fourth roots, you can just take the square root of the square root:

$$T = \sqrt[4]{F/\sigma} = (F/\sigma)^{0.25} = (F/\sigma)^{0.5\times0.5} = ((F/\sigma)^{0.5})^{0.5} = \sqrt{\sqrt{F/\sigma}}$$

Using this formula, calculate the leading and trailing side temperatures:

$$T_{\text{leading}} = \sqrt[4]{F_{\text{leading}}/\sigma} = \sqrt[4]{\frac{14.6 \text{ W m}^{-2}}{5.67 \times 10^{-8} \text{ W m}^{-2}K^{-4}}} = 127 \text{ K}$$

$$T_{\text{trailing}} = \sqrt[4]{F_{\text{trailing}}/\sigma} = \sqrt[4]{\frac{6.0 \text{ W m}^{-2}}{5.67 \times 10^{-8} \text{ W m}^{-2}K^{-4}}} = 102 \text{ K}$$

3 Spectacular impacts

Problem 3A
The ratio of the impact energies is just:

$$E_2/E_1 = \frac{1/2\ m_2 v_2^2}{1/2\ m_1 v_1^2}$$

If the impact velocities are the same, then $v_1 = v_2$ and everything but the masses cancels:

$$E_2/E_1 = \frac{m_2}{m_1} = \frac{1 \times 10^{14}\ \text{kg}}{8 \times 10^{7}\ \text{kg}} = 1.25 \times 10^6$$

So if comet Tempel 1 impacted the Earth, it would release more than a million times the energy of the Tunguska event.

Problem 3B
The graph lists the energy of the Tunguska event as about 13 megatons TNT equivalent energy, so 1.25 million times that would be about 16 million megatons TNT equivalent energy. This is greater than the global catastrophe threshold of 10^4 to 10^7 megatons TNT equivalent energy, so the comet Tempel 1 impact would indeed be a global catastrophe.

Problem 3C
The graph says that impacts with energies greater than the global catastrophe threshold should occur every few million years. Since the last one was more than 60 million years ago, it would seem that we are overdue for a catastrophe. However, these events are random, so we can't use statistics like this to predict when the next one will take place.