Correlation f^n notes

- useful on small scales
- considering objects

have something, prob of something else at $\vec{r}$ dist away?

$$dP = \bar{n}^2(r)\,(1 + \xi(\vec{r}))$$

want $\xi(\vec{r})$ + variance (to get errors)

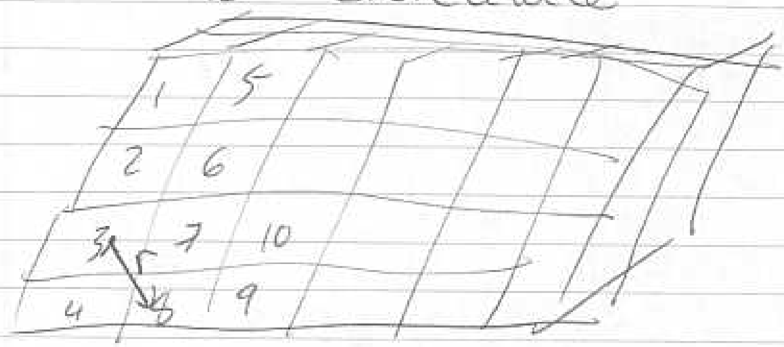
on hand have # gals at positions $\vec{r}_\alpha$

$$n(\vec{r}) = \sum_\alpha \delta(\vec{r} - \vec{r}_\alpha)$$

if $n(\vec{r})$ was continuous density field

$$\langle n(\vec{r})\, n(\vec{r}\,')\rangle = \bar{n}(\vec{r})\,\bar{n}(\vec{r}\,')\,(1 + \xi(\vec{r} - \vec{r}\,'))$$

try to divide up space into patches to calculate



Naive guess doesn't work!

Show it first (WRONG!)

$$\xi(|\vec{r}|) = \frac{\langle N_i N_j \rangle}{\bar{N}^2_{mean}} - 1 \quad -1$$

where patches i, j sep by dist $\vec{r}$

$\langle\ \rangle$ sum over all pairs i,j patches where centers are sep by r

$\bar{N}^2_{meas}$ ~~not~~ actual ξ but measured

$N_i = \#$ in patch $i = \bar{N}\, W_i (1+\delta_i)$

real $\quad$ window fn $\quad$ flucts in i

in 2d $W_i = 0, 1$

$\bar{N}_{meas} = \langle N_i \rangle = \bar{N} \langle W_i (1+\delta_i)\rangle = \bar{N}[\langle W\rangle + \langle \delta W\rangle]$

Consider again

$$\frac{\langle N_i N_j\rangle}{\bar{N}^2_{meas}} - 1 = \frac{\langle N_i N_j\rangle - \bar{N}^2_{meas}}{\bar{N}^2_{meas}}$$

$$= \frac{\langle W_i W_j\rangle + \langle W_i \delta_i W_j\rangle + \langle W_i \delta_j W_j\rangle + \langle W_i \delta_i \delta_j W_j\rangle}{[\langle W\rangle + \langle \delta W\rangle]^2}$$

$$- \frac{[\langle W\rangle + \langle \delta W\rangle]^2}{[\langle W\rangle + \langle \delta W\rangle]^2}$$

$$= \frac{\langle W_i \delta_i \delta_j W_j\rangle}{\langle W\rangle^2} + \text{other stuff:}$$

1) denominator $\langle \delta W\rangle$ term
again, what is $\bar{N}$ really?
2) numerator $\langle \delta W\rangle$, $\langle W \delta W\rangle$ terms

if δ small, $\delta > \delta^2 \rightarrow$ these > signal!

sometimes introd $\quad \bar{\Psi} \sim \langle W \delta W\rangle$
$\bar{\delta} \sim \langle \delta W\rangle$

if had an ∞ volume then
$$\langle W \delta W \rangle \;\&\; \langle \delta W \rangle \to 0$$

but in a "realistically unfair" sample this doesn't happen.

$\langle N \rangle \neq \bar{N}$ too.

Also: for errors, etc, want variance $\sim \langle "\xi^2" \rangle$ $\rightarrow$ extra δ terms squared will not average to zero even in ∞ volume if $\langle \delta \rangle = 0$.

So:

Subtract first & then average roughly
$$\left\langle \frac{(N_i - \langle N \rangle)(N_j - \langle N \rangle)}{\langle N \rangle^2} \right\rangle$$

- error $\sim \delta^2$ not δ
- still wrong $\langle N \rangle$ but . . .

In practice:

often survey has a lot of little patches - complicated window/sel f^{ns}

Way to calculate:

create a "random" catalogue R
- same # pts as original
- same $\langle N \rangle$.

give estimator in terms of correlations between data D & catalog R.

good estimator: "Landy-Szalay" ("essentially" equiv to above)

$$\text{"LS"} = \frac{DD - 2\langle DR \rangle + \langle RR \rangle}{\langle RR \rangle} = \frac{\langle (D-R)^2 \rangle}{\langle RR \rangle}$$

as R is random $\langle RR \rangle = \langle N \rangle^2$

putting in geometry trivial now... just count pts.

Others used in past & now too eg.

$$\frac{\langle DD \rangle \langle RR \rangle}{\langle DR \rangle^2} - 1 \qquad \text{(Hamilton)}$$

Landy & Szalay show has slight bias.

Other issues:

Things people want in estimator.

- low shot noise (LS min in some cases)
- low sensitivity to edge effects (LS mins).

- variance small
  (LS does this)
  if lots of samples w/ sep. r in survey
  central limit thm → gaussian
  $\sqrt{var}$ = 1σ error in ξ
  small var → small errors.
  note this is obs ξ not theoretical one.

- SPEED
  want tractable.
  esp errors - variance.

  - lately - use tree to split up data:
    classify pts if nearby on tree, otherwise don't look.
    can make calc ~ (Num of gals)
  "MKd tree"

- other things
  eg estimator useful for specific correlations (density + seeing)

Refs:

Efstathiou Los houches notes sec. 3.4

Hamilton ApJ '93 417 p. 19
Landy / Szalay ApJ '93 412 p. 64
Szapudi + Szalay ApJ '98 494 L41

(SDSS) astro-ph 0107416 p24-27

also papers on "Spice" by Szapudi
astroph 0107383 fast method.
astro-ph/0110230 Mekd trees