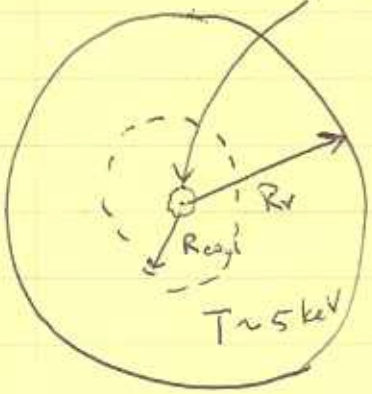
Last time Alexi pointed out that the virial radius of a cluster is where the mean density enclosed is 200 times the critical density

BCG - brightest cluster galaxy



$$R_{vir} \sim 1 - 3\ Mpc.$$

$$\rho_{crit} \simeq \frac{3H^2}{8\pi G}$$

$$\Rightarrow \boxed{n_{crit} \sim 6 \times 10^{-6} \left(\frac{H_0}{70}\right)^2}$$

so $\boxed{\bar{n}_{cluster} \text{ at } R_{vir} \text{ is } = 200\, n_{crit} \simeq 10^{-3} \left(\frac{H}{70}\right)^2}$

The mass of this cluster is

$$\boxed{M \simeq \bar{n}_{crit} \times \frac{4\pi}{3} R_{vir}^3 = 10^{14}\, M_\odot \left(\frac{R_V}{1\,Mpc}\right)^3 \left(\frac{H}{70}\right)^2}$$

so the virial temperature is then

$$\boxed{kT_{vir} \sim \frac{GMm_p}{R_{vir}} \sim 5\ keV \left(\frac{M}{10^{14} M_\odot}\right)\left(\frac{R_{vir}}{1\,Mpc}\right)^{-1}}$$

~~We can~~ This ~~radi~~ temperature is in the X-rays so if it does emit (thermally) clusters should be bright in X-rays

In addition the emission is optically thin. Let's compute the column

$$\boxed{N = \bar{n}_{cluster} \times R_{vir} = 3 \times 10^{21} \text{ cm}^{-2}}$$

which ~~for~~ corresponds to a Thomson depth of

$$\boxed{\tau_{Th} = \sigma_{th} N = 2 \times 10^{-3} \left(\frac{H_0}{70}\right)^2 \left(\frac{R_v}{1 \text{ Mpc}}\right)}$$

So this emission is optically thin in tenuous media and due to <u>Bremstrahlung</u>

so the power of Bremstrahlung per unit volume is

$$\frac{dP}{dV} = 10^{-27} n_p n_e T^{1/2} \text{ ergs cm}^{-3} \text{ s}^{-1}$$

plasma Temperature

# of protons

# of electrons

Note the n^2 dependences. Emission is heavily weighted toward denser regions.

So the cooling will depend heavily on radius

so I will take a straw man's NFW & play with

$$n \sim \frac{1}{R^3} \quad \text{or} \quad n_{baryon} \simeq 0.1\, \bar{n}_{cluster} \left(\frac{R}{R_{vir}}\right)^{-3}$$

$$L \sim \frac{dP}{dV} \times Vol$$

$$\sim 10^{-27}\, T^{1/2}\, n^2\, V^{(R_{vir})} \simeq \boxed{10^{43}\, \frac{\text{ergs}}{s} \left(\frac{R}{R_{vir}}\right)^{-6} \left(\frac{R_{vir}}{1\,Mpc}\right)^{3} \left(\frac{kT}{5\,keV}\right)^{1/2}}$$

The total energy content contained with this volume is

$$E = kT \times nV = \boxed{10^{63} \left(\frac{kT}{5\,keV}\right) \left(\frac{R}{R_{vir}}\right)^{-3} \left(\frac{R_{vir}}{1\,Mpc}\right)^{3}}$$

so at constant T the cooling time is

$$t_{cool} = \frac{E}{L} = 10^{20}\, s = 3 \times 10^{12}\, \text{years} \left(\frac{R}{R_{vir}}\right)^{+3} \left(\frac{kT}{5\,keV}\right)^{1/2}$$

so where does $t_{cool} = t_{Hubble} = 14\,Gyrs$

$$\Rightarrow \left(\frac{R_{cool}}{R_{vir}}\right) = 0.17 \quad \text{or} \quad \boxed{R_{cool} \simeq 170\,kpc \left(\frac{R_{vir}}{1\,Mpc}\right)}$$

~~This is kinda sort what you see~~. ~~show~~ ~~Sanderson figure~~

What is nice is that you can go out and actually measure this fact since we can measure T and L and noting

$$t_{cool} \sim \frac{E}{L} \sim \frac{nT}{n^2 T^{1/2}} \sim \frac{T^{3/4}}{L^{1/2}} \quad \text{as } n \sim \left(\frac{L}{T^{1/2}}\right)^{1/2}$$

so this is roughly what you see. Show Sanderson et al 2006 figure 9.

At the cooling radius, how much mass is inflowing

$$\dot{M} = \frac{M_{gas}}{t_{cool}} = \frac{10^{13} M_\odot}{3 \times 10^{12} \text{ yrs.}} \left(\frac{R}{R_{vir}}\right)^{-3} = 3 M_\odot/\text{yr} \left(\frac{R}{R_{vir}}\right)^{-3}$$

but at $\frac{R}{R_{vir}} = 0.17 \Rightarrow \boxed{\dot{M} \simeq 600 \ M_\odot/\text{yr}}$

This is a huge mass inflow rate. Typically they run from 100 - 200 $M_\odot$/yr but 1000+ $M_\odot$/yr is also seen.

There is <u>some</u> evidence for modest mass inflow

1. nebular emission lines ex NGC 1275 in Perseus cluster
   show Sarazin pg. 113
   and ~~Vot~~ Donahue & Voit 2003 Hubble Hunt.
   - but gas mass is low $\approx M_g \sim 10^7 M_\odot$
2. Star formation - ~~had to be~~ The BCGs show star forming activity but they are hard to reconcile with the total mass flow. ~~ex~~.

Show Cardiel et al 1995 SFR plot

That in a nutshell is the classic cooling flow problem. In addition as Donahue & Voit point out there are radio sources in these cooling flows.
⇒ hints to heating.

The second cooling flow problem.

Below about a few keV ~~tm~~ emission lines start to become very important. But ~~we~~ you need XMM-RGS to get a very high resolution spectra.

We assume that gas cools as follows

i.e. a bunch of cold gas interdispersed with hot gas. So for such an isobaric situation, I expect gas spanning a range of temperatures.

Peterson et al (2003) compute what such a spectrum would look like and got a surprise

Fe L which is an excellent calorimeter of what is going on is missing!

show Peterson & Fabian (2004) figure 8.

Plot as a function of fractional luminosity gives Peterso et al figure 7

So the Peterson et al (2003) result is 2 things
① Violation of the multiphase cooling
② lack of gas below $\sim T_{vir}/3$

One hint I mentioned earlier is that the cooling flow problem might point to some sort of heating

① AGN Feedback.

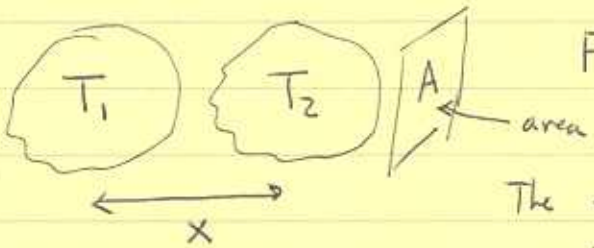
② Conduction

Brian will cover AGN's so I'll leave that up to him.

On to conduction.

First we o-of-magnitude spitzer.

The flux is carried by electrons diffusing through a tenuous gas with density n



$$F = \frac{\Delta E}{A \Delta t}$$

The energy of each blob is

$$E \sim n \underbrace{A x}_{\text{volume}} kT$$

$T_1 > T_2$

$$\Rightarrow \Delta E = n A x k (T_1 - T_2) \quad \cancel{\text{to } T_1 > T_2}$$

The time it takes for electrons to get from one blob to the next is $t_{diff} = t_{coll} \left(\frac{x}{mfp}\right)^2$

diffusion time, collision time, mean free path

The mean free path $mfp = \frac{1}{n\sigma}$

where $\sigma = \pi b^2$ and $kT = \frac{e^2}{b}$

The collision time is $\frac{mfp}{v_{th}}$

so $t_{diff} = \frac{mfp}{v_{th}} \left(\frac{x}{mfp}\right)^2 = \frac{x}{mfp}\frac{x}{v_{th}}$

$$= \frac{n\sigma}{v_{th}} x^2$$

$$\Rightarrow F = \frac{\Delta E}{A\,\Delta t} = \frac{\cancel{n}\,\cancel{A}\,\cancel{x}\,k(T_1 - T_2)}{\cancel{A}\,\cancel{x}\,\frac{\cancel{n}\sigma}{v_{th}}\,x^{\cancel{2}}}$$

$$= \underbrace{\frac{k v_{th}}{\sigma}}\ \underbrace{\frac{\Delta T}{x}}$$

$$F = \frac{k\,(kT)^{5/2}}{e^4 m_e^{1/2}}\frac{dT}{dx} = 2\times10^{-5}\,T^{5/2}\frac{dT}{dx}$$

Spitzer is really 6×10^{-7} so we are off by a factor of 30, which isn't bad.

How much ~~flux~~ luminosity is there?

$$L = 4\pi R^2 F = 4\pi R^2 \; 6\times10^{-7} \; T^{5/2} \frac{T}{R}$$

$$= 4\pi \; 3\times10^{46} \; \frac{\text{ergs}}{\text{s}} \left(\frac{kT}{5\,\text{keV}}\right)^{7/2} \left(\frac{R}{1\,\text{Mpc}}\right)$$

This is 3 orders of magnitude larger than our calculated radiation flux. Which is why conduction was ignored for a long time. i.e. it would heat up all the cold gas.

But there are ways to suppress it namely using magnetic fields

$$R_L = \frac{m_e c}{eB} v_{th} \approx \sqrt{m_e kT}\,\frac{c}{eB} \approx 1.7\times10^{9} \left(\frac{B}{0.1\,\mu G}\right)^{-1} \left(\frac{kT}{5\,\text{keV}}\right)^{\frac{1}{2}} \text{cm}$$

compared to the mfp $= \frac{1}{n\sigma} = \frac{1}{n} \cdot \frac{(kT)^2}{\pi e^4} = 4\times10^{22} \left(\frac{kT}{5\,\text{keV}}\right)^2 \left(\frac{n}{0.01}\right)^{-1}$ cm.

So magnetic field can easily suppress ~~the~~ electron conduction.

- But see more recent work by Zakutsky & Narayan (2005)