AY 202 Assignment 2

due: Tuesday, February 15

Problem 1: Consider, as we did in class, water moving in a tank. The left and right sides of the tank are located at $x = 0$ and $x = L$, respectively, and the tank is infinite in the y-direction. The depth of the water below its equilibrium level ($z = 0$) is h.

Assume now that the water motion is a *standing wave.* That is, the velocity potential ϕ is of the form

$$\phi(x, z, t) = \phi_\circ(z)\, f(x) \cos \omega\, t \ ,$$

and obeys the equation

$$\nabla^2 \phi = 0 \ .$$

(a) What are the boundary conditions to be imposed on $\phi(x, z, t)$?

(b) By applying all your boundary conditions, find expressions for $\phi_\circ(z)$ and $f(x)$.

(c) Find the period $P_\circ$ of the fundamental, or *sloshing*, mode.

(d) Estimate $P_\circ$ numerically for water in your bathtub. Is your answer reasonable?

Problem 2: In lecture, we solved the simple problem of an isothermal atmosphere in a uniform gravitational field $\boldsymbol{g}$. Suppose, instead, that the atmoshpere is isentropic, so that

$$P = K \rho^\gamma \ ,$$

where K and γ are constants.

(a) Derive an expression for the density ρ as a function of height h. Your expression will contain, besides γ, g and h, the molecular weight μ, the gas constant $\mathcal{R}$, and $T_\circ$ and $\rho_\circ$, the temperature and density, respectively, at ground level.

(b) Derive an expression for dT/dh. What is dT/dh (in deg/km) for the Earth's atmosphere?

(c) Do a little research to find out how your prediction compares with reality. What is the main factor accounting for the difference?

Problem 3: C & C, Problem 18

Problem 4: C & C, Problem 22