AY 202 Assignment 3

due: Tuesday, March 1

Problem 1: Consider a viscous, incompressible fluid. with kinematic viscosity ν, that is contained between two rotating, coaxial cylinders of radii R_1 and R_2 ($R_1 < R_2$). The cylinders are very long compared to their radii. If the inner and outer angular rotational speeds are Ω_1 and Ω_2, respectively, what is the steady-state motion of the fluid?

Hint: For any scalar function ψ, the Laplacian $\nabla^2\psi$ in cylindrical coordinates (R, ϕ, z), is given by

$$\nabla^2\psi \;=\; \frac{1}{R}\,\frac{\partial}{\partial R}\left(R\,\frac{\partial\psi}{\partial R}\right) \;+\; \frac{1}{R^2}\,\frac{\partial^2\psi}{\partial\phi^2} \;+\; \frac{\partial^2\psi}{\partial z^2}\;.$$

The components of $\nabla^2\boldsymbol{A}$, where $\boldsymbol{A}$ is any vector, are

$$\begin{aligned}
[\nabla^2\boldsymbol{A}]_R \;&=\; \nabla^2 A_R \;-\; \frac{A_R}{R^2} \;-\; \frac{2}{R^2}\,\frac{\partial A_\phi}{\partial\phi} \\
[\nabla^2\boldsymbol{A}]_\phi \;&=\; \nabla^2 A_\phi \;+\; \frac{2}{R^2}\,\frac{\partial A_R}{\partial\phi} \;-\; \frac{A_\phi}{R^2} \\
[\nabla^2\boldsymbol{A}]_Z \;&=\; \nabla^2 A_Z\;.
\end{aligned}$$

Problem 2: C & C, Problem 14

Problem 3: C & C, Problem 32

Problem 4: C & C, Problem 45