

## AY 202 Assignment 4

due: Tuesday, March 15

**Problem 1:** In class we considered an incompressible fluid of density  $\rho$  that was initially stationary, but contained a line vortex of strength  $\kappa$ , lying along the  $z$ -axis. We found the evolution of the vorticity  $\omega(R, t)$ , where  $R$  is the cylindrical radius. The vorticity diffuses outward, with a diffusion coefficient equal to the kinematic viscosity  $\nu$ .

(a) Now consider the evolving pressure within the fluid. First find analytically  $p(R, 0)$ , the initial pressure distribution. In your expression, let  $p_0$  be the pressure at infinity. Sketch  $p(R, 0)$ .

(b) Next find  $p(R, t)$ . You should be able to express your answer in terms of the exponential function  $E_1(x)$ , where

$$E_1(x) \equiv \int_x^\infty \frac{\exp(-t)}{t} dt .$$

(c) Make several careful sketches of the pressure profiles for various times. If you are ambitious, you may look up tabulated values of  $E_1(x)$  to help in your plots.

**Problem 2:** Shocks generate heat. Here you will determine the jump in  $s$ , the entropy per unit mass, across a viscous shock.

(a) Using the first law of thermodynamics, find an expression for  $s$  of an ideal gas. Your expression should contain  $C_v$  (the heat capacity per unit mass at fixed volume),  $p$ ,  $\rho$ , and  $\gamma$ .

(b) Suppose gas with velocity  $u_1$ , density  $\rho_1$ , etc., crosses a stationary shock front, so that its velocity becomes  $u_2$ , etc. Write an expression for  $s_2 - s_1$  in terms of the pre- and postshock pressures.

(c) Show that a “rarefaction shock,” i.e., one with  $p_2 < p_1$ , violates the second law of thermodynamics. That is,  $s_2 - s_1$  would be negative under these circumstances.

**Problem 3:** C & C, Problem 27

**Problem 4:** C & C, Problem 33