AY 202 Assignment 7

due: Tuesday, May 3

Problem 1: The solar wind consists of hot, ionized gas that gently accelerates outward. Since the fluid velocity is subsonic over a considerable distance, let us explore a slightly different possibility - that this gas is actually in hydrostatic balance. In other words, we hypthesize that the plasma constitutes an extended envelope for the Sun.

(a) Assuming spherical symmetry. write a differential equation for $p(r)$, the pressure distribution in this hypothetical envelope. The gas, which is spatially isothermal with sound speed a_T, is pulled inward by the gravity of the Sun (with mass $M_\odot$), and pushed outward by the radial pressure gradient.

(b) Let $p_\circ$ be the pressure at the Sun's coronal base ($r = r_\circ$), where the envelope begins. By integrating your equation from (a), find an expression for p_∞, the pressure at very large r, in terms of $p_\circ$.

(c) The number density of hydrogen ions at the coronal base is $n_\circ = 3 \times 10^7 \ \mathrm{cm}^{-3}$, and the temperature is $T = 2 \times 10^6$ K. Given that $r_\circ = 1.25\, R_\odot$, find numerically both $p_\circ$ and p_∞ in dyne cm^{-2}.

(d) The asymptotic pressure p_∞ should match p_{ISM}, the pressure of the interstellar medium. For the latter, use a typical HI cloud, with a hydrogen number density of $30 \ \mathrm{cm}^{-3}$ and a temperature of 80 K. How does your calculated p_∞ compare with p_{ISM}? What would happen to the putative static envelope?

Problem 2: Consider the magnetic analog to Problem 1 of Assignment 5. First imagine a static, perfectly conducting gas, permeated by a uniform magnetic field $\boldsymbol{B}_\circ$. This gas has a uniform density $\rho_\circ$.

The fluid is now set into motion, so that the velocity is $\boldsymbol{u}(\boldsymbol{r})$, the new density $\rho(\boldsymbol{r})$, and the new magnetic field $\boldsymbol{B}(\boldsymbol{r})$. The motion is steady-state, and $\rho(\boldsymbol{r})$ and $\boldsymbol{B}(\boldsymbol{r})$ are also time-independent.

(a) Write a relationship between $\boldsymbol{u}$ and $\boldsymbol{B}$ under these circumstances. (*Hint*: Use the ideal MHD equation.)

(b) Assume that the change from the initial, static fluid is relatively small. That is,

$$\begin{aligned}\boldsymbol{B} &= \boldsymbol{B}_\circ + \boldsymbol{B}' \\ \boldsymbol{u} &= \boldsymbol{u}' \\ \rho &= \rho_\circ + \rho' \ ,\end{aligned}$$

where all primed quantities are small. By linearizing your equation from (a), as well as the equation of mass continuity, find an expression for the change in $\boldsymbol{u}'$ in the direction of $\boldsymbol{B}_\circ$.

Problem 3: Here is another problem involving steady-state motion of a magnetized gas. In this case, however, the motion need not be slow. Suppose the gas is swirling around an axis pointing in the z-direction. The fluid motion is axisymmetric. In cylindrical coordinates $(R,\ \phi,\ z)$:

$$\boldsymbol{u} = u_\phi\,(R, z)\,\hat{\boldsymbol{e}}_\phi \ .$$

The embedded magnetic field is also axisymmetric, and strictly poloidal:

$$\boldsymbol{B} = B_R\,(R, z)\,\hat{\boldsymbol{e}}_R + B_z\,(R, z)\,\hat{\boldsymbol{e}}_z \ .$$

(a) The fluid is perfectly conducting, so that ideal MHD applies. Find the change in the quantity u_ϕ/R along the magnetic field. (*Hint*: You will also need $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$.)

(b) Give a physical interpretation of this result.

Problem 4: C & C, Problem 54 (NB: Please use cgs units, as we did in class.)