AY250 Assignment 1

due: Thursday, Sept 9, 2010

1 - We have seen that the molecular line tracer $^{13}C^{16}O$ effectively probes the cloud filaments of Taurus-Auriga. At a typical location, a bit displaced from the central axis of such a filament, the measured hydrogen column density is $N_H = 2 \times 10^{22}$ cm^{-2}. The diameter of the filament is 4 pc, and its length is four times greater. Assume the filaments are being viewed nearly face-on.

(a) What is n_H, the average *volumetric* number density of hydrogen atoms?

(b) Estimate the total mass of the filament.

The stars within the region have mean proper motions of 2 km s^{-1} in each direction. Their typical age is 3×10^6 yr.

(c) Over the stellar lifetime, do you expect the filamentary pattern of stars to be maintained? In formulating your answer, consider whether the stars are truly on ballistic trajectories. (*Hint:* The line width of the $^{13}C^{16}O$ is also about 2 km s^{-1}.)

2 - A star is embedded in a dusty cloud. Spectroscopy reveals it to be a main-sequence object of spectral type A0. Its apparent magnitudes in the B- and V-bands are 14.3 and 12.8, respectively.

(a) What is the object's color excess, $E_{\rm B-V}$? Note that an A0 star, be definition, has zero absolute magnitude in all wavebands.

(b) What is the visual extinction, A_V, to the star?

(c) What is N_H, the intervening column density?

(d) How far away is the star?

(e) By what factor is the K-band flux from the star attenuated on its way to the Earth?

3 - Suppose that the dust opacity κ_λ varies as λ^{-n} from the B to the V band. Here, n is some positive constant.

(a) Predict the value of $R \equiv A_V/E({\rm B-V})$.

(b) Given that the typical observed R is 3.1, what is the effective n in this range of wavelengths? Does this result seem consistent with Figure 2.7?

4 - The distribution of Mathis, Rumpl, and Nordsieck is strongly weighted to smaller grains, for radii a ranging from $a_{\rm min} = 0.005\ \mu$m to $a_{\rm max} = 0.25\ \mu$m.

(a) By what factor does the mean grain size in this range exceed $a_{\rm min}$?

(b) For wavelengths of light less than $a_{\rm min}$, the extinction increases as a^2. Hence, the effective radius is $a_{\rm rms} \equiv \langle a^2 \rangle^{1/2}$, the root-mean-square value. Evaluate $a_{\rm rms}$ numerically.

(c) For wavelengths exceeding $a_{\rm max}$, the extinction increases as a^3. Here, the effective radius is the root-mean-cube, $a_{\rm rmc} \equiv \langle a^3 \rangle^{1/3}$. Evaluate $a_{\rm rmc}$.

5 - While most of the mass in a giant molecular cloud consists of clumps, there is considerable space between these entities.

(a) Using the data in Table 3.1, what is the total volume occupied by all the clumps in a typical giant molecular cloud? (The clumps are listed as "individual dark clouds.") Estimate the *filling factor* of the clumps, *i.e.*, the ratio of their aggregate volume to that of the entire complex.

(b) In reality, the clumps are distributed in mass according to equation (3.1). Assume this distribution holds between a minimum mass of 30 $M_\odot$ and a maximum of $10^3\ M_\odot$. How many clumps does a typical complex contain? What is your more refined estimate of the filling factor?

(c) As a consistency check, estimate the total mass of HI gas that fills the space between clumps.

6 - We have seen how CO observations measure the velocity dispersion within a giant molecular cloud. The linewidth stems from the interclump velocity.

(a) Again using Table 3.1 to gauge typical clump properties, determine $t_{\rm coll}$, the average time for two clumps to collide. Idealize the clumps as uniform-density spheres and assume their cross section is their projected area. For their relative speed, use the virial value. The collision time provides one simple estimate for the duration of the entire complex.

(b) In calculating $t_{\rm coll}$, we have ignored the gravitational attraction of clumps for each other. Because of this *gravitational focusing*, the effective cross section is increased, and $t_{\rm coll}$ is multiplied by a factor $f_{\rm grav}$, less than unity. Write an algebraic expression for $f_{\rm grav}$ in terms of the mass, radius, and relative speed of the clumps. Evaluate this factor numerically.