AY250 Assignment 2

due: Thursday, Sept 23, 2010

1 - (*N.B. Reading Sections 5.6 and 6.3 will be helpful.*) The OH molecule is only useful for measuring $\boldsymbol{B}$ along the line of sight provided the molecule is sufficiently abundant. Let us see why, in dense cores, this issue has limited the available results. Anticipating the discussion in Chapter 7, OH is produced by cosmic ray ionization of molecular hydrogen. The volumetric production rate may be expressed as $p_1 \, \zeta(\mathrm{H}_2) \, n_{\mathrm{H}_2}$. Here, p_1 is a numerical constant, and $\zeta(\mathrm{H}_2)$ (which has units of s^{-1}) is the rate at which cosmic rays ionize each hydrogen molecule.

(a) In diffuse clouds, OH is destroyed by ambient ultraviolet photons. If the characteristic photodissociation time is τ_{photo}, show that the steady-state ratio $n_{\mathrm{OH}}/n_{\mathrm{H}_2}$ is independent of cloud density, a situation that is favorable observationally.

(b) In clouds dense enough that ultraviolet radiation is excluded, OH is destroyed primarily by reaction with ambient atomic hydrogen:

$$\mathrm{OH} + \mathrm{O} \rightarrow \mathrm{O}_2 + \mathrm{H} \ ,$$

where the associated rate constant is $k = 5 \times 10^{-11} \ \mathrm{cm}^3 \ \mathrm{s}^{-1}$. Given that $n_{\mathrm{O}}/n_{\mathrm{H}_2}$ is itself a fixed ratio, show that n_{OH} is now independent of cloud density. Thus, $n_{\mathrm{OH}}/n_{\mathrm{H}_2}$ falls as n_{H_2} rises.

An independent way of measuring $\boldsymbol{B}$ is through the polarization of starlight. The pattern of polarization vectors gives the direction of $B_\perp$, the field component in the plane of the sky. The field is not straight, but has wiggles. These wiggles plausibly arise from cloud turbulence, which we suppose to be an isotropic superposition of hydromagnetic waves. As will be shown in Chapter 9, such waves travel at the Alfvén speed $V_A \equiv \sqrt{B/4\,\pi\,\rho}$, where ρ is the mass density. If $y(x,t)$ is the curve representing a projected field line, then we may write

$$y = y_\circ \cos k\,(x - V_A\,t) \ ,$$

where the wave amplitude $y_\circ$ and wavenumber k are constants.

(c) If $y' \equiv \partial y/\partial x$ and $\dot{y} \equiv \partial y/\partial t$, argue that

$$(y')^2 = (\Delta\theta)^2$$
$$(\dot{y})^2 = \frac{1}{3}\,\Delta V_{\mathrm{turb}}^2 \ .$$

Here, $\Delta\theta$ is the mean angular excursion of a field line, while ΔV_{turb} is the inferred, three-dimensional turbulent velocity.

(d) Derive an expression for $B_\perp$ in terms of ρ, ΔV_{turb}, and $\Delta\theta$.

(e) A portion of the Taurus filaments called B18 is observed to have $\Delta V_{\rm turb} = 1.7$ km s^{-1}, $\Delta\theta = 29°$, and a hydrogen number density $n_{\rm H} = 700$ cm^{-3}. What is $B_\perp$?

2 - In Section 7.1, we saw how the cosmic ray ionization rate of hydrogen can be inferred from the observed abundance of OH, whose production is initiated by this ionization. There are analogous and complementary schemes involving other molecules. Here we outline such a method applied to the deuterated hydrogen molecule HD, which is observed in diffuse clouds irradiated by background massive stars.

(a) After a hydrogen atom is ionized by a cosmic ray proton (see equation (7.6)), the ion undergoes charge exchange with ambient deuterium:

$$\mathrm{H^+ + D \rightarrow D^+ + H}\ .$$

This reaction can proceed in reverse, at a rate found by multiplying the forward one by the Boltzmann factor $\exp(-\Delta E/k_b T)$. Here, $\Delta E \equiv I_D - I_H$, where I_D and I_H are the ionization potentials of deuterium and hydrogen, respectively. These may be obtained from equation (2.1), using the appropriate reduced mass for the electron. What do you conclude about the rates of forward and backward reaction rates in an HI cloud of temperature 100 K?

(b) From your result in (a), what is the steady-state ionization fraction of deuterium atoms, n_{D^+}/n_D, compared to that of hydrogen atoms, n_{H^+}/n_H?

(c) Ionized deuterium creates HD through an ion-molecule reaction:

$$\mathrm{D^+ + H_2 \rightarrow HD + H^+}\ ,$$

whose rate constant (with units of cm^3 s^{-1}) we denote as $k_{\rm DH}$. The molecule HD is destroyed by photodissociation, with a characteristic time $\tau_{\rm photo}$. Write down the steady-state number density of HD in terms of $k_{\rm DH}$, $\tau_{\rm photo}$, and the densities n_{D^+} and $n_{\rm H_2}$.

(d) The $\mathrm{H^+}$ created by cosmic rays is principally destroyed by reaction with atomic oxygen, as in equation (7.8). Let the associated rate constant be $k_{\rm OH}$. Write an expression for $\zeta(HI)$ in terms of the given rate constants, $\tau_{\rm photo}$, and the observed abundance ratios $n_{\rm HD}/n_{\rm H_2}$, n_O/n_H, and n_D/n_H.

3 - Consider the interstellar radiation field, whose mean intensity per logarithmic frequency interval, νJ_ν, is plotted in Figure 7.4.

(a) According to the text, the total energy density of this radiation, integrated over all frequencies, is $u_{\rm rad} = 1.1$ eV cm^{-3}. What is the gas temperature T_g of a hypothetical

HI cloud with $n_H = 100$ cm^{-3} whose thermal energy density matches $u_{\rm rad}$? Compare your answer to the typical temperature of an HI cloud and comment.

(b) Imagine a more opaque molecular cloud bathed in the interstellar field, which is assumed to be isotropic. Let F_ν be the specific flux impinging on a planar section of the cloud. What is $\nu\,F_\nu$ in terms of $\nu\,J_\nu$, the quantity shown in Figure 7.4?

(c) In the wavelength range 10^3 $\mathring{A} < \lambda < 10^4$ $\mathring{A}$, plot $\nu\,F_\nu$ as a function of ν at visual extinctions A_V of 3, 5, and 10 mag. Use the interstellar extinction curve, Figure 2.7, to make these plots

4 - Ultraviolet radiation within HI clouds ejects electrons from grain surfaces. In steady state, this efflux is matched by an influx of ambient electrons colliding with the surface. By balancing these two rates, we may determine Ze, the net charge on the grain.

(a) Consider a spherical grain of radius a and charge Ze. What is the cross section $\sigma(v)$ this object presents to electrons of mass m_e streaming by with speed v?

(b) Suppose the electrons have number density n_e and share the gas temperature T. Then $\mathcal{R}_{\rm in}$, the rate at which they strike a single grain, is given by $n_e\,\langle v\,\sigma(v)\rangle$, where the average is over a Maxwell-Boltzmann distribution. Find an analytic expression for $\mathcal{R}_{\rm in}$.

(c) Show that the outgoing rate of electron emission is

$$\mathcal{R}_{\rm out} \;=\; 4\,\pi^2\,a^2\,y \int \frac{J_\nu}{h\nu}\,d\nu \;.$$

Here, y is the photoemission yield, i.e., the fraction of incoming ultraviolet photons that actually eject an electron. Theory gives $y \approx 0.1$. The quantity J_ν is the mean intensity of the interstellar radiation field (see Fig. 7.4), and the integral is over photon energies from 10 to 13.6 eV.

(d) By equating $\mathcal{R}_{\rm in}$ and $\mathcal{R}_{\rm out}$, make a numerical estimate of Z for a grain of radius $a = 0.1\,\mu$m. You may use the typical HI cloud values $n_e = 0.05$ cm^{-3} and $T = 100$ K.

5 - Within the interstellar medium, heating of the gas occurs primarily through the photoelectric effect on grains, as described quantitatively in Section 7.2. The main cooling mechanism is the emission of the 128 μm fine-structure line of C II.

(a) Calculate analytically and plot the gas temperature T_g as a function of the density n_H in the range $-2 < \log n_H < +3$. Compare your result to Figure 2.5a, and comment on any differences.

(b) Similarly, plot the gas pressure, P/k_B over the same density range. Compare to Figure 2.5b.

(c) Assume the actual pressure and temperature of the interstellar medium is $P/k_B = 3000$ K cm^{-3}. What are your predicted density and temperature of the cold and warm neutral medium? How does your result for the former compare with real HI clouds?

6 - The cooling flux by CO from a molecular cloud is given by $F_{\rm CO}$ in equation (7.34). Let us derive this important result another way, starting from the volumetric cooling rate, $\Lambda^*_{\rm CO}$, in equation (7.35).

(a) Since the cooling radiation is isotropic, $\Lambda^*_{\rm CO}$ is related to the emissivity j_ν by

$$\Lambda^*_{\rm CO} = 4\,\pi\,j_\nu\,\Delta\nu\ ,$$

where $\Delta\nu \equiv \Delta\nu_{J+1,J}$. Now use Kirkhoff's law, equation (2.30), to express $F_{\rm CO} = \pi\,B_\nu(T_g)\Delta\nu$ in terms of $\Lambda^*_{\rm CO}$, the mean mass density ρ, and κ_ν.

(b) If D is the cloud diameter, then we may write

$$\rho\,\kappa_\nu = \frac{\tau_{J+1,J}}{D}\ .$$

Here, $\tau_{J+1,J}$ is given in terms of τ_{10} by equation (7.30). Using equation (7.35) for $\Lambda^*_{\rm CO}$, together with equation (7.28) for τ_{10}, find $F_{\rm CO}$.